UNIT-6

ANGLE MODULATION (FM) – II

Topics: Demodulation of FM waves, Phase Locked Loop, Non-linear Model of the phase locked loop, Linear model of the phase locked loop, FM stereo multiplexing, Nonlinear effects in FM systems, and FM systems.

Frequency demodulation is the process that enables us to recover the original modulating signal from a frequency modulated signal. Frequency Demodulator produces an output signal with amplitude directly proportional to the instantaneous frequency of FM wave.

Frequency demodulators are broadly classified into two categories:

(i) Direct method – examples: frequency discriminators and zero crossing detectors.

(ii) Indirect method – example: phase locked loop.

The direct methods use the direct application of the definition of instantaneous frequency.

The indirect method depends on the use of feed back to track variations in the instantaneous frequency of the input signal.

Slope Circuit:

This is a circuit in which the output voltage is proportional to the input frequency. An example is a differentiator. The output of the differentiator, x(t) = ds(t)/dt and the transfer function, $H(f) = j2\pi f$.



Fig: 6.1 – *A simple differentiator with transfer function.*

Slope detector:

A slope detector circuit consists of two units: a slope circuit and an envelope detector. The slope circuit converts the frequency variations in the FM signal into a voltage signal, which resembles an AM signal. The envelope circuit obtains the output signal proportional to the message signal.



Fig: 6.2 – Block diagram of a Slope detector.

Consider an FM signal as defined below: (equation 6.1)

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right], \quad \text{where} \quad f_i(t) = f_c + k_f m(t) \tag{6.1}$$

Let the slope circuit be simply differentiator:

$$s_{1}(t) = -A_{c} \left[2\pi f_{c} + 2\pi k_{f} m(t) \right] \sin \left[2\pi f_{c} t + 2\pi k_{f} \int_{0}^{t} m(\tau) d\tau \right]$$
$$s_{o}(t) \approx -A_{c} \left[2\pi f_{c} + 2\pi k_{f} m(t) \right]$$

The output of the slope circuit is thus proportional to the message signal, m(t).

A simple slope detector circuit is shown if fig-6.3, consists of slope circuit and an envelope circuit.



Fig: 6.3 – A simple Slope detector circuit.

Balanced Frequency Discriminator: The frequency discriminator consists of slope circuits and envelope detectors. An ideal slope circuit is characterized by the transfer function that is purely imaginary, varying linearly with frequency inside a prescribed frequency interval. The transfer function defined by the equation 6.2 and is shown in figure 6.4a.



Fig: 6.4 – (a) Frequency response of ideal slope circuit, H₁(f).
(b) Frequency response of complex low pass filter equivalent
(c) Frequency response of ideal slope circuit complementary to part(a).

Consider an FM signal s(t) having spectrum from (fc $-B_T/2$) to (fc $+ B_T/2$) and zero outside this range. Let $s_1(t)$ be the output of the slope circuit. Any slope circuit can be considered as an equivalent low pass filter driven with the complex envelope of input, FM wave.

Let $H_1^{(f)}(f)$ is complex transfer function of the slope circuit. This function is related to $H_1(f)$ by $\tilde{H}_1(f - f_c) = H_1(f), \text{ for } f > 0 \qquad \dots (6.3)$

Using equations 6.2 and 6.3, we get

$$\tilde{H}_1(f) = \begin{cases} j2\pi a \left(f + \frac{B_T}{2} \right) &, \quad -\frac{B_T}{2} \le f \le \frac{B_T}{2} \\ 0 &, \quad \text{otherwise} \end{cases} \qquad \dots(6.4)$$

This is depicted in the figure 6.4b.

Let s(t) be the FM wave, defined in equation(6.1) and its complex envelope be $\tilde{s}(t)$ given by:

complex envelope of
$$s(t)$$

 $\tilde{s}(t) = A_c \exp\left[j2\pi k_f \int_0^t m(t)dt\right]$
(6.5)

Let $s_1(t)$ be the complex envelope of the response of the slope circuit defined by fig-6.4a.

The Fourier transform of $s_1(t)$ is

$$\therefore \widetilde{S}_{1}(f) = \widetilde{H}_{1}(f) \widetilde{S}(f)$$

$$= \begin{cases} j2\pi a \left(f + \frac{B_{T}}{2}\right) \widetilde{S}(f), & -\frac{B_{T}}{2} \le f \le \frac{B_{T}}{2} \\ 0, & \text{otherwise} \end{cases}$$
(6.6)

Using the differentiation in time domain property of Fourier transform;

$$\therefore \quad \tilde{s}_{1}(t) = a \left[\frac{d\tilde{s}(t)}{dt} + j\pi B_{T}\tilde{s}(t) \right]$$
$$= j\pi B_{T}aA_{c} \left[1 + \frac{2k_{f}}{B_{T}}m(t) \right] \exp \left[j2\pi k_{f} \int_{0}^{t}m(t)dt \right] \quad (6.7)$$

Therefore the response of the slope circuit is defined as (6.8):

$$s_{1}(t) = \operatorname{Re}\left[\tilde{s}_{1}(t)\exp(j2\pi f_{c}t)\right]$$
$$= \pi B_{T}aA_{c}\left[1 + \frac{2k_{f}}{B_{T}}m(t)\right]\cos\left[2\pi f_{c}t + 2\pi k_{f}\int_{0}^{t}m(t)dt + \frac{\pi}{2}\right] \qquad (6.8)$$

The signal, $s_1(t)$ is a hybrid modulated wave in which both the amplitude and frequency of the carrier wave vary the message signal, m(t).

If
$$\left|\frac{2k_f}{B_T}m(t)\right| < 1$$
 for all t, resulting output of envelope detector is
 $\left|\tilde{s}_1(t)\right| = \pi B_T a A_c \left[1 + \frac{2k_f}{B_T}m(t)\right]$ (6.9)

The bias term $(\pi B_T a A_c)$ is proportional to the slope 'a' of the transfer function of the slope circuit. The bias may be removed by subtracting from the envelope detector output from the output of a second envelope detector preceded by a complementary slope circuit with a transfer function H₂(f) as described in fig:6.4c.

$$\tilde{H}_2(f) = -\tilde{H}_1(-f) \qquad (6.10)$$

Let $s_2(t)$ be the output of the complementary slope circuit produced by the incoming FM wave s(t). The envelope of the circuit is

$$\left|\tilde{s}_{2}(t)\right| = \pi B_{T} a A_{c} \left[1 - \frac{2k_{f}}{B_{T}} m(t)\right]$$
(6.11)

The difference between the two envelopes in equations (6.9) and (6.11) is

$$s_0(t) = |\tilde{s}_1(t)| - |\tilde{s}_2(t)| = 4\pi k_f a A_c m(t)$$
 (6.12)

Thus an ideal frequency discriminator can be modelled as a pair of slope circuits followed by envelope detectors and a summer as shown in fig-6.5 which is called balanced frequency discriminator.



Fig: 6.5 – Idealized model of Balanced frequency discriminator.

The idealized model can be closely realized using the circuit shown in fig-6.6 which consists of two resonant circuits. The upper and lower resonant filters are tuned to frequencies above and below the un-modulated carrier frequency, f_c . The amplitude responses of the tuned filters and the total response are shown in the fig-6.7.



Fig: 6.6 – Balanced frequency discriminator.

The overall performance of the balanced frequency discriminator will be good only when both the filters have high Q-factor and a proper frequency separation between the tuning frequencies of the two filters. However there will be distortions present in the output due to the following factors:

- 1. Spectrum of FM wave is not exactly zero for the frequencies outside the range.
- 2. Tuned filters are not strictly band-limited
- 3. RC filters in the envelope detector introduce distortions
- 4. Tuned filter characteristics are not linear over the whole frequency band.



Fig: 6.7 – Frequency response.

Zero Crossing Detector:

This detector circuit exploits the property that the instantaneous frequency of an FM wave is approximately given by

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = \frac{1}{2\Delta t} \qquad (6.13)$$

where Δt is the time difference between the adjacent zero crossings of the FM wave. Let n_0 be the number of zero crossings inside an interval T and then express the time Δt is given by

$$\Delta t = \frac{T}{n_0} and f_i(t) = \frac{1}{2\Delta t} = \frac{n_0}{2T} \quad (6.14)$$

Thus the instantaneous frequency is linearly related to the message signal m(t); it can be recovered from the knowledge of n_0 .

A simplified block diagram of a zero crossing detector based on the above principle is shown in the fig-6.8. The basic blocks are Limiter, Pulse generator and Integrator. The Limiter produces a square wave version of the input FM wave. The Pulse generator produces pulses at the positive going edges and negative going edges of the limiter output. The Integrator performs the averaging over the interval T and hence produces the message signal.



Phase-Locked Loop (PLL):

The PLL is a negative feedback system that consists of three major components: a multiplier, a loop filter and a voltage controlled oscillator (VCO) connected in the form of a loop as shown in the fig-6.9. The VCO is a sine wave generator whose frequency is determined by the voltage applied to it.



Fig: 6.9 – Phase Locked Loop

Initially the VCO is adjusted such that control voltage is zero, two conditions are satisfied:

- 1. Frequency of VCO is set precisely at the carrier frequency, fc and
- 2. VCO output has a 90° phase shift w.r.t. to the un-modulated carrier wave.

Let the input signal applied to the PLL be s(t):

$$s(t) = A_c \sin \left[2\pi f_c t + \phi_1(t) \right]$$

where $\phi_1(t) = 2\pi k_f \int_0^t m(t) dt$ (6.15)

Let the VCO output be r(t):

$$r(t) = A_{v} \cos \left[2\pi f_{c}t + \phi_{2}(t) \right] . \qquad \dots (6.16)$$

where $\phi_{2}(t) = 2\pi k_{v} \int_{0}^{t} v(t) dt \qquad \dots (6.17)$

where k_v is the frequency sensitivity of VCO in Hz/volt.

Incoming FM signal and VCO output are applied to the multiplier. The Multiplier output $e(t) = 2 k_m s(t) r(t)$, produces two components: (where k_m – multiplier gain in volt⁻¹)

1. A high frequency component:

$$k_m A_c A_v \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)]$$

2. A low frequency component:

$$k_m A_c A_v \sin[\phi_1(t) - \phi_2(t)]$$

The high frequency component is eliminated by the loop filter (low pass type) and VCO.

Then the input to the Loop filter is:

$$e(t) = k_m A_c A_v \sin[\phi_1(t) - \phi_2(t)]$$
(6.18)
$$e(t) = k_m A_c A_v \sin[\phi_e(t)]$$
....(6.19)

The Phase error :
$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

= $\phi_1(t) - 2\pi k_v \int_0^t v(t) dt$ (6.20)

The loop filter operates on the input e(t) to produce the output: v(t) = e(t) * h(t); where h(t) is impulse response of the loop filter.

$$v(t) = \int_{-\infty}^{\infty} e(\tau)h(t-\tau)d\tau \quad \dots (6.21)$$

Differentiating the equation 6.20; we get

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi k_v v(t)$$

Using 6.21 and 6.19; and rearranging

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_0 \int_{-\infty}^{\infty} \sin\left[\phi_e(\tau)\right] h(t-\tau) d\tau \quad \dots (6.22)$$

where $K_0 = k_m k_v A_c A_v$: loop - gain parameter

The equation (6.22) represents a Non-linear model of a phase locked loop and is shown in fig-6.10. The nonlinear model resembles the basic PLL structure. The Multiplier is replaced by a subtractor and a sinusoidal nonlinearity, and the VCO by an integrator. But the nonlinear model is very difficult to analyze.



Fig: 6.10 – Non-linear model of a Phase Locked Loop

Linear Model of Phase-Locked Loop :

The nonlinear model is a complicated model and is difficult to analyze. Hence a simpler version of this model called a linear model is obtained.

An assumption that phase error, $\phi_e(t)$ is small leads to an approximation:

$$sin[\phi_e(t)] = \phi_e(t)$$
 (6.23)

and substituting in 6.19; we get

$$e(t) = k_m A_c A_v \phi_e(t)$$
 (6.24)

Thus the nonlinear model in fig-6.10 can be written in a simplified form as shown in fig:6.11.



Fig: 6.11 –Linear model of a Phase Locked Loop

Substituting equation (6.23) in (6.22) leads to

$$\frac{d\phi_{e}(t)}{dt} = \frac{d\phi_{1}(t)}{dt} - 2\pi K_{0} \int_{-\infty}^{\infty} [\phi_{e}(\tau)]h(t-\tau)d\tau$$
$$\frac{d\phi_{e}(t)}{dt} + 2\pi K_{0} [\phi_{e}(t) * h(t)] = \frac{d\phi_{1}(t)}{dt} \qquad \dots (6.25)$$

Applying Fourier transform to equation 6.25;

$$j2\pi f \phi_e(f) + 2\pi K_0 \phi_e(f) H(f) = j2\pi f \phi_1(f)$$

$$\Phi_{e}(f) = \frac{jf}{jf + K_{0}H(f)}\Phi_{1}(f) \qquad \dots (6.26)$$

Let L(f) be open-loop transfer function of PLL.

$$L(f) = K_0 \frac{H(f)}{jf}$$
(6.27)

Then $\phi_e(f)$ can be written as

$$\Phi_{e}(f) = \frac{1}{1 + L(f)} \Phi_{1}(f) \qquad \dots \dots (6.28)$$

The output of the linear model in fig 6.11, v(t) can be defined as

$$v(t) = \frac{K_0}{k_v} \Phi_e(t) * h(t) \qquad \dots (6.29)$$

Applying Fourier transform and using (6.27) we get

$$V(f) = \frac{K_0}{k_v} H(f) \Phi_e(f) = \frac{jf}{k_v} L(f) \Phi_e(f)$$

u sin g (6.28)
if $L(f)$

$$V(f) = \frac{Jf}{k_{\nu}} \frac{L(f)}{1 + L(f)} \Phi_1(f) \quad \dots \quad (6.30)$$

As the loop transfer function, L(f) approaches large value, $\phi_e(f)$ approaches zero. The phase of VCO becomes asymptotically equal to the phase of the incoming wave and <u>phase-lock</u> is established.

For
$$|L(f)| \gg 1$$
, $V(f) \cong \frac{jf}{k_{\nu}} \Phi_1(f)$

Applying inverse Fourier transform to the above equation:

$$v(t) \cong \frac{1}{2\pi k_v} \frac{d\phi_1(t)}{dt} \quad \dots \quad (6.31)$$

Substituting for $\phi_1(t)$ from (6.15)

$$v(t) \cong \frac{k_f}{k_v} m(t) \qquad \dots \qquad (6.32)$$

Thus the output of the linearized model is proportional to the message signal.

FM Stereo Multiplexing:

Stereo multiplexing is a form of frequency division multiplexing designed to transmit two separate signals via the same carrier. It is widely used in the FM radio broadcasting to send two different elements of a program. For example the different elements can be sections of orchestra, a vocalist and an accompanist. This gives a spatial dimension to its perception for the listener at the receiving end.

The two important factors that influence the FM stereo transmission are:

- 1. The transmission has to operate within the allocated FM broadcast channels.
- 2. It has to compatible with the monophonic receivers.

The FM stereo transmitter consists of a multiplexing system. The block diagram of the multiplexer is shown in fig-6.12.



Fig: 6.12 – Multiplexer system in transmitter of FM stereo.

Let $m_l(t)$ and $m_r(t)$ denote the two signals from the two different microphones at the transmitter end of the system. They are applied to a matrixer that generates the sum signal and the difference signal. The sum signal $[m_l(t)+m_r(t)]$ is used in the base band form only. The difference signal $[m_l(t) - m_r(t)]$ along with a 38 kHz sub-carrier are applied to a product modulator to generate a DSBSC modulated wave. The sub- carrier is generated from a frequency doubler using 19 kHz oscillator. The three signals: sum signal, difference signal and a pilot carrier signal of frequency 19 kHz are combined/added to obtain the multiplexed signal. The multiplexed signal can be defined as:

$$m(t) = [m_l(t) + m_r(t)] + [m_l(t) - m_r(t)] \cos(4\pi f_c t) + K \cos(2\pi f_c t) \quad \dots \quad (6.33)$$

where $f_c=19$ kHz and K is a constant, chosen to maintain the pilot between 8% and 10% of the peak frequency deviation. The spectrum of the multiplexed signal is shown in the fig6.13.



Fig: 6.13 - Spectrum of Stereo multiplexed signal, m(t).

The multiplexed signal is used as a modulating signal for the FM modulator to produce an FM signal for transmission.

The FM stereo receiver consists of demodulator and de-multiplexer. The de-multiplexer is shown in fig.6.14. The de-multiplexer consists of three filters: a low pass filter and two band pass filters. The base band low pass filter recovers the sum signal in the band 0-15 kHz, the band pass filter recovers the modulated difference signal in the band 23-53 kHz and a narrow band pass filter is used to recover the pilot carrier 19 kHz. The sub-carrier signal of frequency 38 kHz is obtained by frequency doubling the pilot carrier. The difference signal is then obtained from sub-carrier and modulated difference signal using a coherent detector. The matrixer (add/subtractor) reconstructs the left signal, $m_l(t)$ and the right signal, $m_r(t)$.



Fig: 6.14 : De-multiplexer system in receiver of FM stereo.

Non-linear Effects in FM Systems:

Nonlinearities are present in all electrical networks in one form or the other. The two basic forms of nonlinearities are: Strong non-linearity and weak non-linearity.

- The Strong non-linearity is introduced intentionally and in a controlled manner for some specific application. Ex: Square-law Modulators, Limiters, and Frequency Multipliers.
- 2. The Weak non-linearity is of parasitic nature which arises due to imperfections. The effect of this is to limit the useful signal levels in a system.

In this section the effects of channel on FM signal is considered. Let $v_i(t)$ be an FM signal;

$$v_i(t) = A_c \cos[2\pi f_c t + \phi(t)] \dots (6.34)$$

Consider a communication channel whose transfer characteristic are defined by the non-liner input output relation: ($v_i(t)$ be the input and $v_o(t)$ be the output of the channel)

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t)$$
 (6.35)

 $v_{o}(t) = a_{1} A_{c} \cos[2\pi f_{c} t + \phi(t)] + a_{2} A_{c}^{2} \cos^{2}[2\pi f_{c} t + \phi(t)] + a_{3} A_{c}^{3} \cos^{3}[2\pi f_{c} t + \phi(t)]$

Expanding and simplifying, we get

$$v_{0}(t) = \frac{1}{2}a_{2}A_{c}^{2} + \left(a_{1}A_{c} + \frac{3}{4}a_{3}A_{c}^{3}\right)\cos\left[2\pi f_{c}t + \phi(t)\right] + \frac{1}{2}a_{2}A_{c}^{2}\cos\left[4\pi f_{c}t + 2\phi(t)\right] + \frac{1}{4}a_{3}A_{c}^{3}\cos\left[6\pi f_{c}t + 3\phi(t)\right] \qquad \dots (6.36)$$

The first term represents the DC component, the remaining three terms represent the frequency modulated signals with carrier frequencies f_c , $2f_c$ and $3f_c$.

To extract the FM signal from the channel output, it is necessary to separate the FM signal with carrier f_c from the closest component of carrier frequency $2f_c$. Let Δf denote the frequency deviation and W denote the highest frequency component of message signal, m(t). The channel output $v_o(t)$ will have the highest component from first FM as $(f_c + \Delta f + W)$ and the lowest component from the second FM is $(2f_c - (2\Delta f + W))$. Hence the necessary condition required to separate the FM signal is:

$$(2f_c - (2\Delta f + W)) > (f_c + \Delta f + W)$$

Therefore $f_c > 3\Delta f + 2W$ (6.37)

Thus by using a band pass filter of mid frequency f_c and bandwidth ($2\Delta f + 2W$) the channel output is reduced to

.

$$\therefore v_0(t) \bigg|_{\substack{f = f_c \\ BW = 2\Delta f + 2w}} = \left(a_1 A_c + \frac{3}{4} a_3 A_c^3 \right) \cos[2\pi f_c t + \phi(t)] \quad \dots (6.38)$$

Therefore the effect of passing an FM signal through a channel with amplitude nonlinearities, followed by appropriate filtering, is simply to modify its amplitude. Hence unlike AM, FM signals are less affected by the channel distortions.

The FM signal is sensitive to phase nonlinearities. Phase nonlinearities are encountered in Microwave radio systems, which is due to the phase characteristics of repeaters or amplifiers used in the system.

Thus the FM signal is less sensitive to amplitude nonlinearities and more sensitive to phase nonlinearities.

FM System:

A FM system consists of two basic units: FM transmitter and FM receiver. A little information is added in this section about both transmitter and receiver.

FM Transmitter

The basic block diagram of FM transmitter is shown in fig.6.15. The audio input signal is first pre-processed using pre-emphasis circuit to minimize the variation in SNR (signal-to-noise ratio) at the receiver. The narrow band modulator is generated and then passed through frequency multipliers to get wide band FM. The wide band FM signal is fed to driver and then to a power amplifier which increases the signal strength before transmission.

Pre-emphasis: Most common analog signals have high frequency components that are relatively low in amplitude than low frequency ones. Ambient electrical noise is uniformly distributed. Therefore, the SNR for high frequency components is lower compared to low

frequency components. To correct this problem, input signal is *pre-emphasized* before frequency modulation. In FM broadcasting, the high frequency components are boosted by passing the modulating signal through a HPF with a 75 ms time constant before modulation. This topic is discussed in detail in noise analysis of FM systems.



Fig: 6.15: A simple FM transmitter

FM Receiver:

A receiver based on super heterodyne principle is shown in the fig.6.16. The RF amplifier boosts the signal strength of received weak signal. The mixer and local oscillator together performs the down conversion of FM signal to IF range. The discriminator circuit performs the demodulation process to detect the message signal.



Fig: 6.16: A simple FM receiver

Amplitude Limiter: An FM wave gets distorted when it is transmitted through the channel. The FM wave at the input of the receiver will not have constant amplitude because of the channel imperfections. Hence at the receiver it is necessary to remove the amplitude variations prior to the demodulation. This is carried out by an amplitude limiter circuit in the receiver.

An amplitude limiter circuit consists of a hard limiter and a band pass filter. The hard limiter will convert distorted FM wave into an FM square wave. The band pass filter will block all the harmonics in FM square wave to obtain an undistorted FM wave.

De-emphasis: At the FM receiver, the signal after demodulation must be *de-emphasized* by a filter with complementary characteristics as the pre-emphasis filter to restore the relative amplitudes of the modulating signal. The de-emphasis network is a simple R-C network similar to a low pass filter.

Summary: In this chapter various demodulation schemes such frequency discriminator, PLL based demodulators are discussed. In addition to that the FM stereo multiplexing method is explained. During the transmission FM signal the channel will induce distortions due to the nonlinearities. To minimize these proper choice of filters are essential. In the end, a discussion is done on the FM transmitter and FM receivers.