UNIT-5

ANGLE MODULATION (FM) – I

Topics: Basic definitions, FM, narrow band FM, wide band FM, transmission bandwidth of FM waves, and generation of FM waves: indirect FM and direct FM.

Angle modulation is a method of analog modulation in which either the phase or frequency of the carrier wave is varied according to the message signal. In this method of modulation the amplitude of the carrier wave is maintained constant.

- ***Angle Modulation is a method of modulation in which either Frequency or Phase of the carrier wave is varied according to the message signal.***

In general form, an angle modulated signal can represented as

$$s(t) = A_c \cos[\theta(t)] \qquad ...(5.1)$$

Where A_c is the amplitude of the carrier wave and $\theta(t)$ is the angle of the modulated carrier and also the function of the message signal.

The instantaneous frequency of the angle modulated signal, s(t) is given by

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \qquad ...(5.2)$$

The modulated signal, s(t) is normally considered as a rotating phasor of length A_c and angle $\theta(t)$. The angular velocity of such a phasor is $d\theta(t)/dt$, measured in radians per second.

An un-modulated carrier has the angle $\theta(t)$ defined as

$$\theta(t) = 2\pi f_c t + \phi_c \qquad .....(5.3)$$

Where f_c is the carrier signal frequency and ϕ_c is the value of $\theta(t)$ at t = 0.

The angle modulated signal has the angle, $\theta(t)$ defined by

$$\theta(t) = 2\pi f_c t + \phi(t) \qquad .....(5.4)$$

There are two commonly used methods of angle modulation:

1. Frequency Modulation, and
2. Phase Modulation.

Phase Modulation (PM):

In phase modulation the angle is varied linearly with the message signal m(t) as :

$$\theta(t) = 2\pi f_c t + k_p m(t) \qquad .....(5.5)$$

where k_p is the phase sensitivity of the modulator in radians per volt.

Thus the phase modulated signal is defined as

$$s(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right] \qquad ...(5.6)$$

Frequency Modulation (FM):

In frequency modulation the instantaneous frequency $f_i(t)$ is varied linearly with message signal, m(t) as:

$$f_i(t) = f_c + k_f m(t) \qquad ....(5.7)$$

where k_f is the frequency sensitivity of the modulator in hertz per volt.

The instantaneous angle can now be defined as

$$\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)dt \qquad ....(5.8)$$

and thus the frequency modulated signal is given by

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right] \qquad ....(5.9)$$

The PM and FM waveforms for the sinusoidal message signal are shown in the fig-5.1.

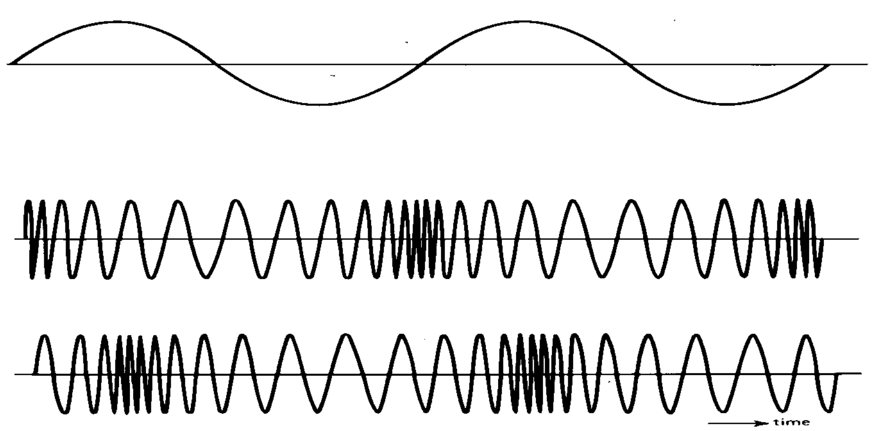

Fig: 5.1 – PM and FM Waveforms with a message signal

Example 5.1:

Find the instantaneous frequency of the following waveforms:

(a) $S_1(t) = Ac \ Cos\ [100\pi\ t + 0.25\ \pi\ ]$

(b) $S_2(t) = Ac \ Cos\ [100\pi\ t + \sin(20\ \pi\ t)\ ]$

(c) $S_3(t) = Ac \ Cos\ [100\pi\ t + (\pi\ t^2)\ ]$

Solution: Using equations (5.1) and (5.2):

(a) $f_i(t) = 50$ Hz; Instantaneous frequency is constant.

(b) $f_i(t) = 50 + 10 \cos(20\ \pi\ t)$; Maximum value is 60 Hz and minimum value is 40 Hz.

Hence, instantaneous frequency oscillates between 40 Hz and 60 Hz.

(c) $f_i(t) = (50 + t)$

The instantaneous frequency is 50 Hz at t=0 and varies linearly at 1 Hz/sec.

Relation between Frequency Modulation and Phase Modulation:

A frequency modulated signal can be generated using a phase modulator by first integrating m(t) and using it as an input to a phase modulator. This is possible by considering FM signal as phase modulated signal in which the modulating wave is integral of m(t) in place of m(t). This is shown in the fig-5.2(a). Similarly, a PM signal can be generated by first differentiating m(t) and then using the resultant signal as the input to a FM modulator, as shown in fig-5.2(b).

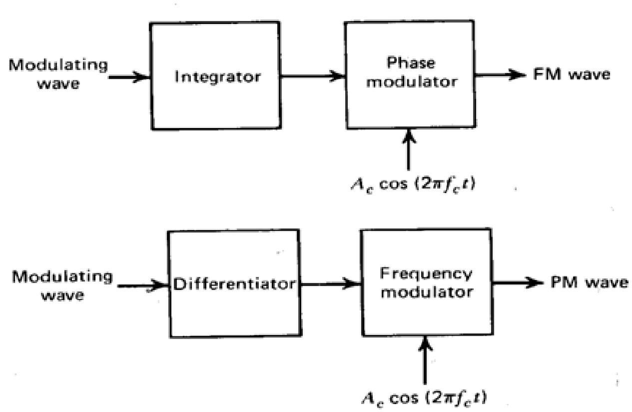

Fig: 5.2 – Scheme for generation of FM and PM Waveforms

Single-Tone Frequency Modulation:

Consider a sinusoidal modulating signal defined as:

$$m(t) = A_m \, Cos(\, 2\pi f_m \, t) \qquad \text{…. (5.10)}$$

Substituting for m(t) in equation (5.9), the instantaneous frequency of the FM signal is

$$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t) \;= f_c + \Delta f \cos(2\pi f_m t)$$

where Δf is called the frequency deviation given by $\Delta f = k_f A_m$ ..... (5.11a)

and the instantaneous angle is

$$\begin{aligned}\theta(t) &= 2\pi \int_0^t f_i(t) dt \\ &= 2\pi f_c t + \frac{\Delta f}{f_m} \sin(\, 2\pi f_m t) \\ &= 2\pi f_c t + \beta \sin(\, 2\pi f_m t) \qquad \text{....(5.11b)}\end{aligned}$$

$$\text{where } \beta = \frac{\Delta f}{f_m}; \text{ modulation index}$$

The resultant FM signal is

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \qquad \text{....(5.12)}$$

The frequency deviation factor indicates the amount of frequency change in the FM signal from the carrier frequency f_c on either side of it. Thus FM signal will have the frequency components between (f_c - Δf) to (f_c +Δf). The modulation index, β represents the phase deviation of the FM signal and is measured in radians. Depending on the value of β, FM signal can be classified into two types:

1. Narrow band FM (β << 1) and
2. Wide band FM (β >> 1).

Example-5.2: A sinusoidal wave of amplitude 10volts and frequency of 1 kHz is applied to an FM generator that has a frequency sensitivity constant of 40 Hz/volt. Determine the frequency deviation and modulating index.

Solution: Message signal amplitude, A_m = 10 volts, Frequency f_m = 1000 Hz and the frequency sensitivity, k_f = 40 Hz/volt.

Frequency deviation, $\Delta f = k_f A_m = 400$ Hz

Modulation index, $\beta = \Delta f / f_m = 0.4$, (indicates a narrow band FM).

Example-5.3: A modulating signal m(t) =10 Cos(10000πt) modulates a carrier signal, A_c Cos(2πf_ct). Find the frequency deviation and modulation index of the resulting FM signal. Use k_f = 5kHz/volt.

Solution: Message signal amplitude, A_m = 10 volts, Frequency f_m = 5000 Hz and the frequency sensitivity, k_f = 5 kHz/volt.

Frequency deviation, $\Delta f = k_f A_m = 50$ kHz

Modulation index, $\beta = \Delta f / f_m = 10$, (indicates a wide band FM).

Frequency Domain Representation of Narrow Band FM signal:

Expanding the equation (5.12) using trigonometric identities,

$$\begin{aligned} s(t) &= A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \\ &= A_c \cos(2\pi f_c t)\cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t)\sin[\beta \sin(2\pi f_m t)] \end{aligned}$$

For NBFM, (β << 1), we can approximate,

$$\cos[\beta \sin(2\pi f_m t)] \approx 1 \;\; and \;\; \sin[\beta \sin(2\pi f_m t)] \approx \beta \sin(2\pi f_m t)$$

Hence, $$s(t) = A_c \cos(2\pi f_c t) - A_c \beta \sin(2\pi f_c t)\, \sin(2\pi f_m t) \qquad .....(5.13)$$

Using trigonometric relations;

$$s(t) = A_c \cos(2\pi f_c t) + \frac{A_c \beta}{2} \left[\cos(2\pi (f_c + f_m) t) - \cos(2\pi (f_c - f_m) t\right] \quad ..(5.14)$$

The above equation represents the NBFM signal. This representation is similar to an AM signal, except that the lower side frequency has negative sign. The magnitude spectrum of NBFM signal is shown in fig-5.3, which is similar to AM signal spectrum. The bandwidth of the NBFM signal is $2f_m$, which is same as AM signal.

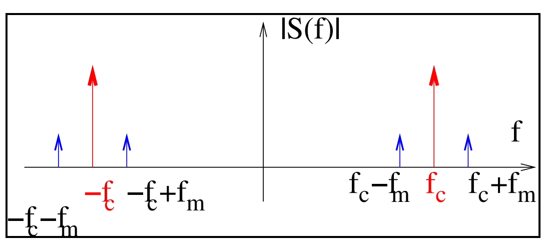

Fig: 5.3 - Magnitude Spectrum of NBFM Waveform.

Frequency Domain Representation of Wide-Band FM signals:

The FM wave for sinusoidal modulation is given by

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$
$$= A_c \cos(2\pi f_c t)\cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t)\sin[\beta \sin(2\pi f_m t)]$$

The FM wave can be expressed in terms of complex envelope as:

$$s(t) = \mathrm{Re}[A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))]$$
$$= \mathrm{Re}[\tilde{s}(t)\exp(j2\pi f_c t)] \qquad \ldots (5.15)$$

The complex envelope of the FM wave

$$\tilde{s}(t) = A_c \exp[j\beta \sin(2\pi f_m t)] \; and \; \tilde{s}(t) : \text{periodic function with } f_m$$

The complex envelope is a periodic function of time, with a fundamental frequency equal to the modulation frequency f_m. The complex envelope can be expanded in the form of complex series:

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp[j2\pi n f_m t] \qquad \ldots (5.16)$$

The complex Fourier coefficient, c_n equals,

$$c_n = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \tilde{s}(t) \exp(-j2\pi n f_m t) dt$$
$$= f_m A_c \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \exp[j\beta \sin(2\pi f_m t) - j2\pi n f_m t] dt \qquad \ldots (5.17)$$

Substituting x = ($2\pi f_m t$), in the above equation we can rewrite

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp(j(\beta \sin x - nx)) dx \qquad \ldots (5.18)$$

The n[th] order Bessel function of the first kind is defined as

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp(j(\beta \sin x - nx)) dx \qquad \ldots (5.19)$$

Comparing equations (5.18) and (5.19), we get $C_n = A_c J_n(\beta)$

Substituting in (5.16), the complex envelope is

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j2\pi n f_m t) \qquad \ldots (5.20)$$

Substituting in (5.15), the FM signal can be written as

$$s(t) = A_c \operatorname{Re}\left[\sum_{n=-\infty}^{\infty} J_n(\beta)\exp\left[j2\pi\left(f_c + nf_m\right)t\right]\right]$$

$$\mathrm{s(t)} = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos\left[2\pi\left(f_c + nf_m\right)t\right] \qquad \ldots(5.21)$$

The above equation is the Fourier series representation of the single tone FM wave. Applying the Fourier transform to (5.21),

$$S(f) = \frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta\left(f - f_c - nf_m\right) + \delta\left(f + f_c + nf_m\right)\right] \qquad \ldots(5.22)$$

The spectrum S(f) is shown in fig-5.4. The above equation indicates the following:

(i) FM signal has infinite number of side bands at frequencies ($f_c \pm nf_m$).

(ii) Relative amplitudes of all the spectral lines depends on the value of $J_n(\beta)$.

(iii) The number of significant side bands depends on the modulation index (β). With (β<<1), only $J_0(\beta)$ and $J_1(\beta)$ are significant. But for (β>>1), many sidebands exists.

(iv) The average power of an FM wave is $P = 0.5A_c^2$ (based on Bessel function property).

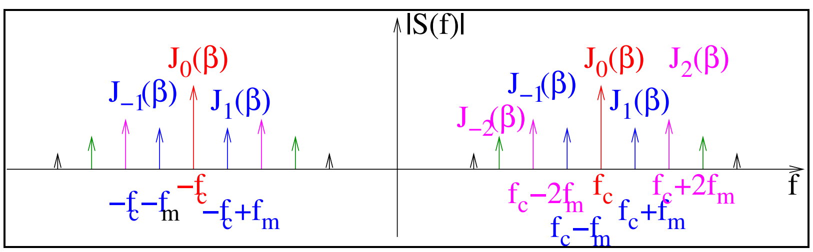

Fig: 5.4 - Magnitude Spectrum of Wide Band FM Wave.

Bessel's Function:

Bessel function is an useful function to represent the FM wave spectrum. The general plots of Bessel functions are shown in fig-5.5 and table (5.1) gives the values for Bessel function coefficients. Some of the useful properties of Bessel functions are given below:

(a) $J_n(\beta) = (-1)^n J_{-n}(\beta)$ for all n (5.23a)

(b) $J_{n+1}(\beta) + J_{n-1}(\beta) = \frac{2n}{\beta} J_n(\beta)$ (5.23b)

(c) $\sum_{n=-\infty}^{\infty} J_n^{\,2}(\beta) = 1$ (5.23c)

(d) For smaller values of β, $J_0(\beta) \cong 1,\ J_1(\beta) \cong \frac{\beta}{2}$ *and* $J_n(\beta) \cong 0,\ for\ n > 2$

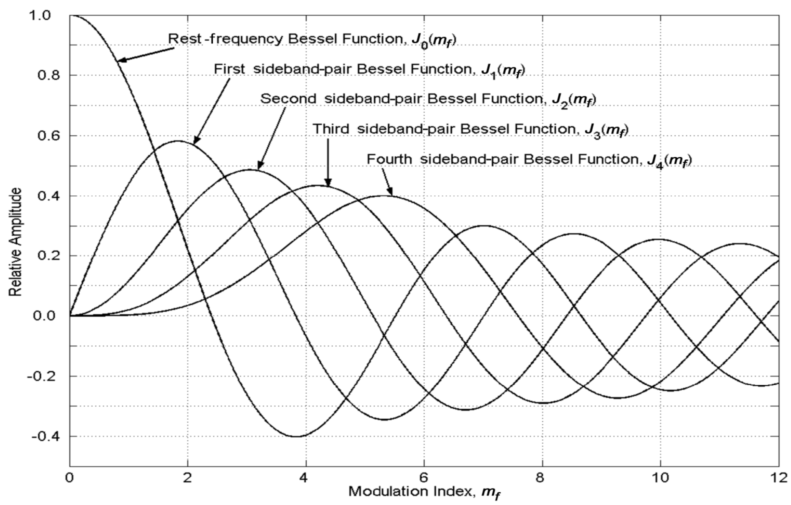

Fig: 5.5 – Plots of Bessel functions

x	*Bessel-function order, n*																
m_f	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.41	0	0.52	0.43	0.20	0.06	0.02	—	—	—	—	—	—	—	—	—	—	—
2.5	–.05	0.50	0.45	0.22	0.07	0.02	0.01	—	—	—	—	—	—	—	—	—	—
3.0	–.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	–.40	–.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	–.18	–.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
5.53	0	–.34	–.13	0.25	0.40	0.32	0.19	0.09	0.03	0.01	—	—	—	—	—	—	—
6.0	0.15	–.28	–.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	–.30	–.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	–.11	–.29	–.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—	—	—
8.65	0	0.27	0.06	–.24	–.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02	—	—	—	—
9.0	–.09	0.25	0.14	–.18	–.27	–.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01	—	—	—
10.0	–.25	0.04	0.25	0.06	–.22	–.23	–.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01	—	—
12.0	0.05	–.22	–.08	0.20	0.18	–.07	–.24	–.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01

Table: 5.1

The Spectrum of FM signals for three different values of β are shown in the fig-5.6. In this spectrum the amplitude of the carrier component is kept as a unity constant. The variation in the amplitudes of all the frequency components is indicated.

For β = 1, the amplitude of the carrier component is more than the side band frequencies as shown in fig-5.6a. The amplitude level of the side band frequencies is decreasing. The dominant components are ($f_c \pm f_m$) and ($f_c \pm 2f_m$). The amplitude of the frequency components ($f_c \pm nf_m$) for n>2 are negligible.

For β = 2, the amplitude of the carrier component is considered as unity. The spectrum is shown in fig-5.6b. The amplitude level of the side band frequencies is varying. The amplitude levels of the components ($f_c \pm f_m$) and ($f_c \pm 2f_m$) are more than carrier frequency component; whereas the amplitude of the component ($f_c \pm 3f_m$) is lower than the carrier amplitude. The amplitude of frequency components ($f_c \pm nf_m$) for n>3 are negligible.

The spectrum for β = 5, is shown in fig-5.6c. The amplitude of the carrier component is considered as unity. The amplitude level of the side band frequencies is varying. The amplitude levels of the components ($f_c \pm f_m$), ($f_c \pm 3f_m$), ($f_c \pm 4f_m$) and ($f_c \pm 5f_m$), are more than carrier frequency component; whereas the amplitude of the component ($f_c \pm 2f_m$) is lower than the carrier amplitude. The amplitude of frequency components ($f_c \pm nf_m$) for n>8 are negligible.

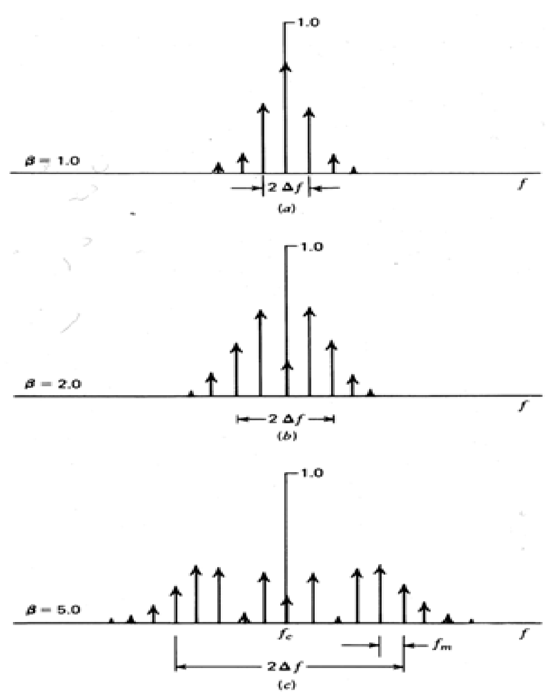

Fig: 5.6 – Plots of Spectrum for different values of modulation index.
(Amplitude of carrier component is constant at unity)

Example-5.4:

An FM transmitter has a power output of 10 W. If the index of modulation is 1.0, determine the power in the various frequency components of the signal.

Solution: The various frequency components of the FM signal are

f_c, $(f_c \pm f_m)$, $(f_c \pm 2f_m)$, $(f_c \pm 3f_m)$, and so on.

The power associated with the above frequency components are: (Refer (5.21))

$(J_0)^2$, $(J_1)^2$, $(J_2)^2$, and $(J_3)^2$ respectively.

From the Bessel function Table, for $\beta = 1$;

$J_0 = 0.77$, $J_1 = 0.44$, $J_2 = 0.11$, and $J_3 = 0.02$

Let $P = 0.5(A_c)^2 = 10$ W.

Power associated with f_c component is $P_0 = P\,(J_0)^2 = 10\,(0.77)^2 = 5.929$ W.

Similarly, $P_1 = P\,(J_1)^2 = 10\,(0.44)^2 = 1.936$ W.

$P_2 = P\,(J_2)^2 = 10\,(0.11)^2 = 0.121$ W.

$P_3 = P\,(J_3)^2 = 10\,(0.02)^2 = 0.004$ W.

Note: Total power in the FM wave,

$$P_{total} = P_0 + 2P_1 + 2P_2 + 2P_3$$

$$= 5.929 + 2(1.936) + 2(.121) + 2(.004) = 10.051 \text{ W}$$

Example-5.5:

A 100 MHz un-modulated carrier delivers 100 Watts of power to a load. The carrier is frequency modulated by a 2 kHz modulating signal causing a maximum frequency deviation of 8 kHz. This FM signal is coupled to a load through an ideal Band Pass filter with 100MHz as center frequency and a variable bandwidth. Determine the power delivered to the load when the filter bandwidth is:

(a) 2.2 kHz (b) 10.5 kHz (c) 15 kHz (d) 21 kHz

Ans: Modulation index, $\beta = 8\text{ k} / 2\text{ k} = 4$;

From the Bessel function Table- 5.1; for $\beta = 4$;

$J_0 = -0.4$, $J_1 = -0.07$, $J_2 = 0.36$, $J_3 = 0.43$, $J_4 = 0.28$, $J_5 = 0.13$, $J_6 = 0.05$, $J_7 = 0.02$

Let $P = 0.5(A_c)^2 = 100$ W and

$P_0 = P(J_0)^2 = 100\,(-0.4)^2 = 16$ Watts.

$P_1 = P(J_1)^2 = 100\,(-0.07)^2 = 0.490$ W.

$P_2 = P(J_2)^2 = 100\,(0.36)^2 = 12.960$ W.

$P_3 = P(J_3)^2 = 100\,(0.43)^2 = 18.490$ W.

$P_4 = P(J_4)^2 = 100\,(0.28)^2 = 7.840$ W.

$P_5 = P(J_5)^2 = 100\,(0.13)^2 = 1.690$ W.

$P_6 = P(J_6)^2 = 100\,(0.05)^2 = 0.250$ W.

(a) Filter Bandwidth = 2.2 kHz

The output of band pass filter will contain only one frequency component f_c.

Power delivered to the load, $P_d = P_0 = 16$ Watts.

(b) Filter Bandwidth = 10.5 kHz

The output of band pass filter will contain the following frequency components:

f_c, $(f_c \pm f_m)$, and $(f_c \pm 2f_m)$

Power delivered to the load, $P_d = P_0 + 2P_1 + 2P_2 = 42.9$ Watts.

(c) Filter Bandwidth = 15 kHz

The output of band pass filter will contain the following frequency components:

f_c, $(f_c \pm f_m)$, $(f_c \pm 2f_m)$, and $(f_c \pm 3f_m)$,

Power delivered to the load, $P_d = P_0 + 2P_1 + 2P_2 + 2P_3 = 79.9$ Watts.

(d) Filter Bandwidth = 21 kHz

The output of band pass filter will contain the following frequency components:

f_c, $(f_c \pm f_m)$, $(f_c \pm 2f_m)$, $(f_c \pm 3f_m)$, $(f_c \pm 4f_m)$, and $(f_c \pm 5f_m)$,

Power delivered to the load, $P_d = P_0 + 2P_1 + 2P_2 + 2P_3 + 2P_4 + 2P_5 = 98.94$ Watts.

Example-5.6:

A carrier wave is frequency modulated using a sinusoidal signal of frequency f_m and amplitude A_m. In a certain experiment conducted with f_m=1 kHz and increasing A_m, starting from zero, it is found that the carrier component of the FM wave is reduced to

zero for the first time when A_m=2 volts. What is the frequency sensitivity of the modulator? What is the value of A_m for which the carrier component is reduced to zero for the second time?

Ans: The carrier component will be zero when its coefficient, $J_0(\beta)$ is zero.

From Table 5.1: $J_0(x) = 0$ for x= 2.44, 5.53, 8.65.

$\beta = \Delta f / f_m = k_f A_m / f_m$ and $k_f = \beta f_m / A_m = (2.40)(1000) / 2 = 1.22$ kHz/V

Frequency Sensitivity, $k_f = 1.22$ kHz/V

The carrier component will become zero for second time when $\beta = 5.53$.

Therefore, $A_m = \beta f_m / k_f = 5.53 (1000) / 1220 = 4.53$ volts

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Transmission Bandwidth of FM waves:

An FM wave consists of infinite number of side bands so that the bandwidth is theoretically infinite. But, in practice, the FM wave is effectively limited to a finite number of side band frequencies compatible with a small amount of distortion. There are many ways to find the bandwidth of the FM wave.

1. Carson's Rule: In single–tone modulation, for the smaller values of modulation index the bandwidth is approximated as $2f_m$. For the higher values of modulation index, the bandwidth is considered as slightly greater than the total deviation $2\Delta f$. Thus the Bandwidth for sinusoidal modulation is defined as:

$$\begin{aligned} B_T &\cong 2\Delta f + 2 f_m = 2\Delta f \left(1 + \frac{1}{\beta}\right) \\ &= 2(\beta + 1) f_m \end{aligned} \qquad (5.24)$$

For non-sinusoidal modulation, a factor called Deviation ratio (D) is considered. The deviation ratio is defined as the ratio of maximum frequency deviation to the bandwidth of message signal.

Deviation ratio , D = (Δf / W), where W is the bandwidth of the message signal and the corresponding bandwidth of the FM signal is,

$$B_T = 2(D + 1) W \qquad \text{... (5.25)}$$

2. Universal Curve : An accurate method of bandwidth assessment is done by retaining the maximum number of significant side frequencies with amplitudes greater than 1% of the un-modulated carrier wave. Thus the bandwidth is defined as "*the 99 percent bandwidth of an FM wave as the separation between the two frequencies beyond which none of the side-band frequencies is greater than 1% of the carrier amplitude obtained when the modulation is removed*".

Transmission Bandwidth - **$BW = 2\, n_{max}\, f_m$,** (5.26)

where f_m is the modulation frequency and 'n' is the number of pairs of side-frequencies such that $|J_n(\beta)| > 0.01$. The value of n_{max} varies with modulation index and can be determined from the Bessel coefficients. The table 5.2 shows the number of significant side frequencies for different values of modulation index.

The transmission bandwidth calculated using this method can be expressed in the form of a universal curve which is normalised with respect to the frequency deviation and plotted it versus the modulation index. (Refer fig-5.7).

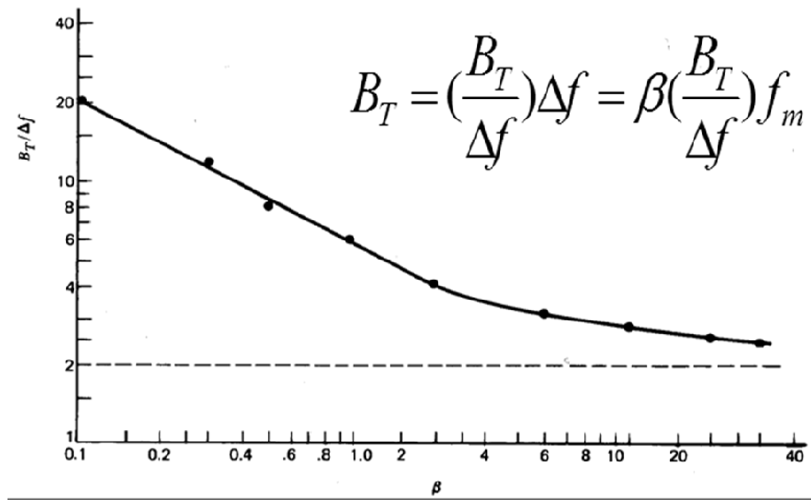
Table 5.2

Number of Significant Side Frequencies of a Wide-band FM Signal for Varying Modulation Index

Modulation Index β	Number of Significant Side Frequencies $2n_{max}$
0.1	2
0.3	4
0.5	4
1.0	6
2.0	8
5.0	16
10.0	28
20.0	50
30.0	70

From the universal curve, for a given message signal frequency and modulation index the ratio (B/ Δf) is obtained from the curve. Then the bandwidth is calculated as:

$$B_T = (\frac{B_T}{\Delta f})\Delta f = \beta(\frac{B_T}{\Delta f})f_m \qquad ...(5.27)$$



Universal Curve for evaluating the bandwidth of FM wave

Fig: 5.7 – Universal Curve

Example-5.7:

Find the bandwidth of a single tone modulated FM signal described by

$S(t)=10 \cos[2\pi 10^8 t + 6 \sin(2\pi 10^3 t)]$.

Solution: Comparing the given s(t) with equation-(5.12) we get

Modulation index, β = 6 and Message signal frequency, f_m = 1000 Hz.

By *Carson's rule* (equation - 5.24),

Transmission Bandwidth, $B_T = 2(\beta + 1) f_m$

$B_T = 2(7)1000 = 14000\ Hz = 14$ kHz

Example-5.8:

Q. A carrier wave of frequency 91 MHz is frequency modulated by a sine wave of amplitude 10 Volts and 15 kHz. The frequency sensitivity of the modulator is 3 kHz/V.
(a) Determine the approximate bandwidth of FM wave using Carson's Rule.
(b) Repeat part (a), assuming that the amplitude of the modulating wave is doubled.
(c) Repeat part (a), assuming that the frequency of the modulating wave is doubled.

Solution: (a) Modulation Index, $\beta = \Delta f / f_m = k_f A_m / f_m = 3\text{x}10/15 = 2$

By Carson's rule; Bandwidth, $B_T = 2(\beta + 1)\, f_m = 90$ kHz

(b) When the amplitude, Am is doubled,

New Modulation Index, $\beta = \Delta f / f_m = k_f A_m / f_m = 3x20/15 = 4$

Bandwidth, $B_T = 2(\beta+1)f_m = 150$ kHz

(c) when the frequency of the message signal, fm is doubled

New Modulation Index, $\beta = 3x10/30 = 1$

Bandwidth, $B_T = 2(\beta+1)f_m = 120$ kHz.

Example-5.9:

Q. Determine the bandwidth of an FM signal, if the maximum value of the frequency deviation Δf is fixed at 75kHz for commercial FM broadcasting by radio and modulation frequency is W= 15 kHz.

Solution: Frequency deviation, $D = (\Delta f / W) = 5$

Transmission Bandwidth, $B_T = 2(D + 1)\, W = 12x15$ kHz = 180 kHz

Example-5.10:

Q. Consider an FM signal obtained from a modulating signal frequency of 2000 Hz and maximum Amplitude of 5 volts. The frequency sensitivity of modulator is 2 kHz/V. Find the bandwidth of the FM signal considering only the significant side band frequencies.

Solution: Frequency Deviation = 10 kHz

Modulation Index, $\beta = \Delta f / f_m = k_f A_m / f_m = 5$;

From table –(5.2) ; $2n_{max} = 16$ for $\beta = 5$,

Bandwidth, $B_T = 2\, n_{max}\, f_m = 16x2$ kHz = 32 kHz.

Example-5.11: A carrier wave of frequency 91 MHz is frequency modulated by a sine wave of amplitude 10 Volts and 15 kHz. The frequency sensitivity of the modulator is 3 kHz/V. Determine the bandwidth by transmitting only those side frequencies with amplitudes that exceed 1% of the unmodulated carrier wave amplitude. Use *universal curve* for this calculation.

Solution:

Frequency Deviation, $\Delta f = 30$ kHz

Modulation Index, $\beta = 3x10/15 = 2$

From the *Universal curve*; for $\beta = 2$; $(B / \Delta f) = 4.3$

Bandwidth, $B = 4.3\ \Delta f = 129$ kHz

Generation of FM Waves:

There are two basic methods of generating FM waves: *indirect method and direct method*. In indirect method a NBFM wave is generated first and frequency multiplication is next used to increase the frequency deviation to the desired level. In direct method, the carrier frequency is directly varied in accordance with the message signal. To understand the indirect method it is required to know the generation of NBFM waves and the working of frequency multipliers.

Generation of NBFM wave:

A frequency modulated wave is defined as: (from equation 5.9)

$$s_1(t) = A_C \cos[2\pi f_C t + \phi_1(t)] \qquad ....(5.28)$$

Where $\phi_1(t) = 2\pi k_1 \int_0^t m(t)dt$

$$s_1(t) = A_C \cos(2\pi f_C t) \cos[\phi_1(t)] - A_C \sin(2\pi f_C t) \sin[\phi_1(t)]$$

Assuming $\phi_1(t)$ is small, then using $\cos[\phi_1(t)] = 1$ and $\sin[\phi_1(t)] = \phi_1(t)$.

$$s_1(t) = A_C \cos(2\pi f_C t) - A_C \sin(2\pi f_C t).[\phi_1(t)]$$

$$s_1(t) = A_C \cos(2\pi f_C t) - 2\pi k_1 A_C \sin(2\pi f_C t).\int_0^t m(t)dt \qquad ...(5.29)$$

The above equation defines a narrow band FM wave. The generation scheme of such a narrow band FM wave is shown in the fig.(5.8). The scaling factor, $(2\pi k_1)$ is taken care of by the product modulator. The part of the FM modulator shown inside the dotted lines represents a *narrow-band phase modulator*.

The narrow band FM wave, thus generated will have some higher order harmonic distortions. This distortions can be limited to negligible levels by restricting the modulation index to $\beta < 0.5$ radians.

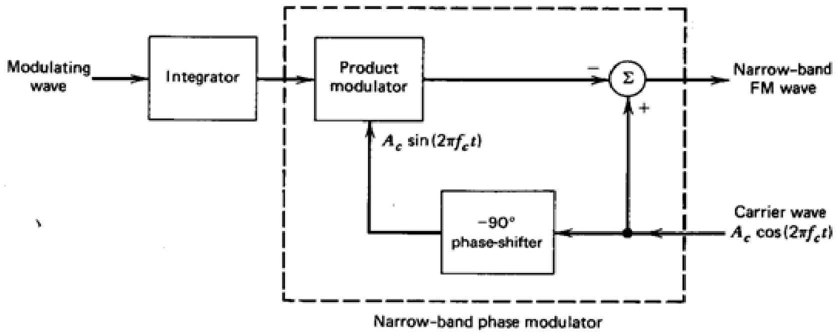


Fig: 5.8 – Scheme to generate a NBFM Waveform.

Frequency Multiplier:

The frequency multiplier consists of a nonlinear device followed by a band-pass filter. The nonlinear device used is a memory less device. If the input to the nonlinear device is an FM wave with frequency, f_c and deviation, Δf_1 then its output *v(t)* will consist of dc component and 'n' frequency modulated waves with carrier frequencies, f_c, $2f_c$, $3f_c$, …… nf_c and frequency deviations a Δf_1, $2\Delta f_1$, $3\Delta f_1$, ... $n\Delta f_1$ respectively.

The band pass filter is designed in such a way that it passes the FM wave centered at the frequency, nf_c with frequency deviation $n\Delta f_1$ and to suppress all other FM components. Thus the frequency multiplier can be used to generate a wide band FM wave from a narrow band FM wave.

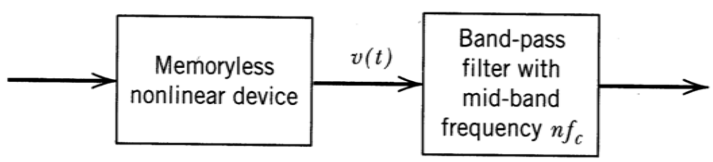


Fig: 5.9 – Frequency Multiplier

Generation of WBFM using Indirect Method:

In indirect method a NBFM wave is generated first and frequency multiplication is next used to increase the frequency deviation to the desired level. The narrow band FM wave is generated using a narrow band phase modulator and an oscillator. The narrow band FM wave is then passed through a frequency multiplier to obtain the wide band FM wave, as shown in the fig:(5.9). The crystal controlled oscillator provides good frequency stability. But this scheme does not provide both the desired frequency deviation and carrier frequency at the same time. This problem can be solved by using multiple stages of frequency multiplier and a mixer stage.

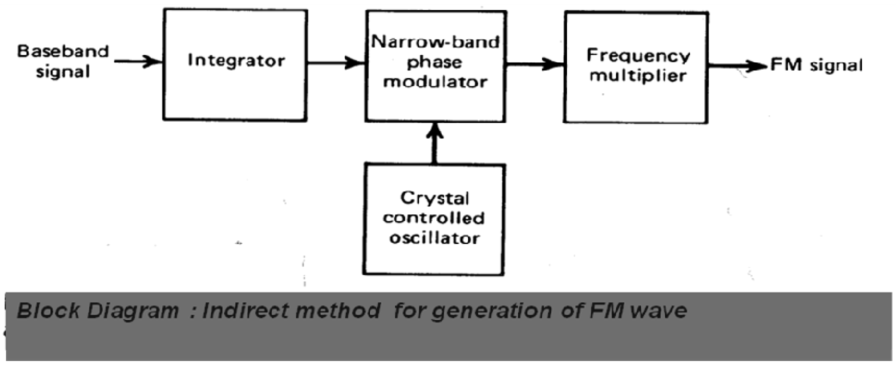


Fig: 5.9 – Generation of WBFM wave

Generation of WBFM by Armstrong's Method:

Armstrong method is an indirect method of FM generation. It is used to generate FM signal having both the desired frequency deviation and the carrier frequency. In this method, two-stage frequency multiplier and an intermediate stage of frequency translator is used, as shown in the fig:(5.10). The first multiplier converts a narrow band FM signal into a wide band signal. The frequency translator, consisting of a mixer and a crystal controlled oscillator shifts the wide band signal to higher or lower frequency band. The second multiplier then increases the frequency deviation and at the same time increases the center frequency also. The main design criteria in this method are the selection of multiplier gains and oscillator frequencies. This is explained in the following steps.

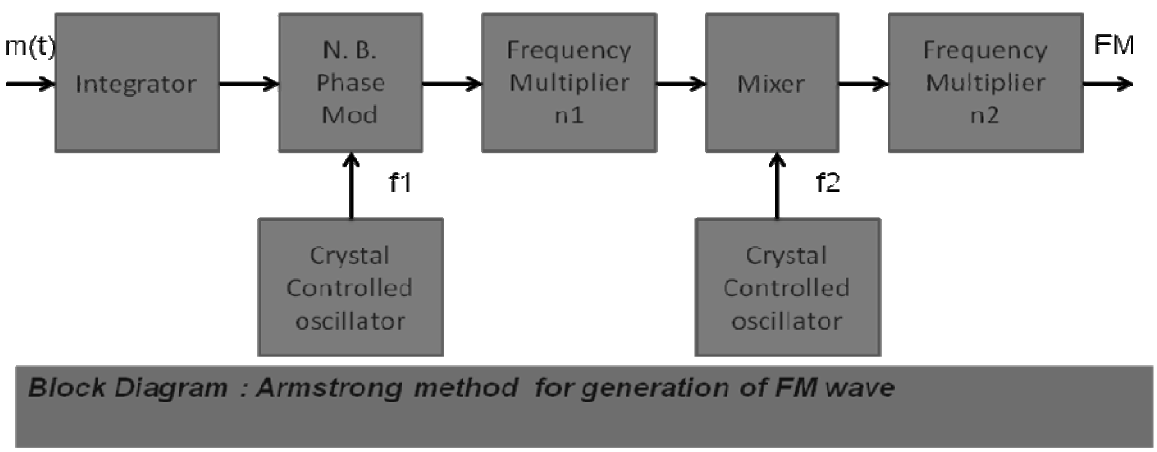


Fig: 5.10 – Generation of WBFM wave by Armstrong method

Design Steps:

Q: How to choose n_1 and n_2 for the given specifications?

1. Select the value of $\beta < 0.5$ for the narrow band phase modulator. This value limits the harmonic distortion by NBPM to minimum.

2. The requirement is that the frequency deviation produced by the lowest modulation frequencies is raised to required Δf. So choose the frequency deviation of NBFM, Δf_1 by selecting the minimum value of f_m.

$$\Delta f_1 = \beta f_{m\,(min)}. \quad \text{---- (a)}$$

3. Frequency Multipliers change the frequency deviation. Hence the total change in the frequency deviation is product of the two deviations:

$$n_1.n_2 = \Delta f / \Delta f_1 \quad \text{----- (b)}$$

4. Frequency Translator (mixer & oscillator) will not change the frequency deviation, it only shifts the FM signal to either upwards and downwards in the spectrum. The output of mixer is

   For down ward translation: $f_c = n_2 (f_2 - n_1. f_1)$ ---- (c)

   and for upward translation: $f_c = n_2 (n_1. f_1 - f_2)$.

5. Choose suitable value for f_2 and solve the equations (b) and (c) simultaneously to find the multiplying factors $n_1$and n_2.

Example 5.12: Design Armstrong FM generator for the generation of WBFM signal with Δf = 75 kHz and f_c = 100 MHz, using the narrow band carrier as 100 kHz and second carrier as 9.5 MHz. Find the suitable multiplying factors. Assume the message signal is defined in the range, 100Hz ~ 15KHz.

Solution:

1. Let phase deviation be, $\beta = 0.2$

2. Frequency deviation of NBFM, $\Delta f_1 = \beta f_{m\,(min)} = 0.2 \times 100 = 20$ Hz

3. $n_1.n_2 = \Delta f / \Delta f_1 = 75000/20 = 3750$ (A)

4. Let f_1 and f_2 be 0.1MHz and 9.5 MHz and f_c is given as 100 MHz.

   $f_c = (f_2 - n_1. f_1)n_2$;

   $100\text{ M} = (9.5\text{ M} - n_1. 0.1\text{M})n_2$ (B)

Solving the equations (A) and (B) simultaneously we get $n_1 = 75$ and $n_2 = 50$.

Generation of WBFM using Direct Method:

In direct method of FM generation, the instantaneous frequency of the carrier wave is directly varied in accordance with the message signal by means of an voltage controlled oscillator. The frequency determining network in the oscillator is chosen with high quality factor (Q-factor) and the oscillator is controlled by the incremental variation of the reactive components in the tank circuit of the oscillator. A *Hartley Oscillator* can be used for this purpose.

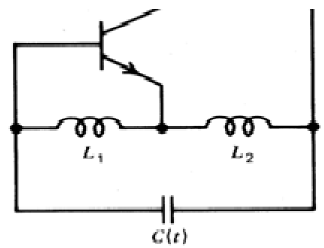


Fig: 5.11 – Hartley Oscillator (tank circuit) for generation of WBFM wave.

The portion of the tank circuit in the oscillator is shown in fig:5.11. The capacitive component of the tank circuit consists of a fixed capacitor shunted by a voltage-variable capacitor. The resulting capacitance is represented by C(t) in the figure. The voltage variable capacitor commonly called as varactor or varicap, is one whose capacitance depends on the voltage applied across its electrodes. The *varactor diode* in the reverse bias condition can be used as a voltage variable capacitor. The larger the voltage applied across the diode, the smaller the transition capacitance of the diode.

The frequency of oscillation of the Hartley oscillator is given by:

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)c(t)}} \qquad \ldots(5.30)$$

Where the L_1 and L_2 are the inductances in the tank circuit and the total capacitance, c(t) is the fixed capacitor and voltage variable capacitor and given by:

$$c(t) = c_0 + \Delta c \cos(2\pi f_m t) \qquad \ldots(5.31)$$

Let the un-modulated frequency of oscillation be f_0. The instantaneous frequency $f_i(t)$ is defined as:

$$f_i(t) = f_0\left[1 + \frac{\Delta c}{c_0}\cos(2\pi f_m t)\right]^{-\frac{1}{2}} \qquad \ldots(5.32)$$

$$\text{where } f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)c_0}} \qquad ...(5.33)$$

$$\therefore f_i(t) = f_0\left[1+\frac{\Delta c}{c_0}\cos(2\pi f_m t)\right]^{-\frac{1}{2}}$$

$$\cong f_0\left[1-\frac{\Delta c}{2c_0}\cos(2\pi f_m t)\right]$$

Thus the instantaneous frequency $f_i(t)$ is defined as:

$$\therefore f_i(t) \cong f_0 + \Delta f\cos(2\pi f_m t) \qquad ...(5.34)$$

The term, Δf represents the frequency deviation and the relation with Δc is given by:

$$\left(\frac{\Delta c}{2c_0} = -\frac{\Delta f}{f_0}\right) \qquad ... (5.35)$$

Thus the output of the oscillator will be an FM wave. But the direct method of generation has the disadvantage that the carrier frequency will not be stable as it is not generated from a highly stable oscillator.

Generally, in FM transmitter the frequency stability of the modulator is achieved by the use of an auxiliary stabilization circuit as shown in the fig.(5.12).

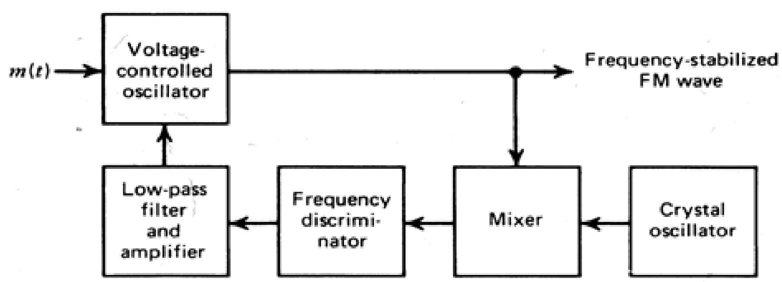


Fig: 5.12 – Frequency stabilized FM modulator.

The output of the FM generator is applied to a mixer together with the output of crystal controlled oscillator and the difference is obtained. The mixer output is applied to a frequency discriminator, which gives an output voltage proportional to the instantaneous frequency of the FM wave applied to its input. The discriminator is filtered by a low pass filter and then amplified to provide a dc voltage. This dc voltage is applied to a voltage controlled oscillator (VCO) to modify the frequency of the oscillator of the FM generator. The deviations in the transmitter carrier frequency from its assigned value will cause a change in the dc voltage in a way such that it restores the carrier frequency to its required value.

Advantages and disadvantages of FM over AM:

Advantages of FM over AM are:

1. Less radiated power.
2. Low distortion due to mproved signal to noise ratio (about 25dB) w.r.t. to man made interference.
3. Smaller geographical interference between neighbouring stations.
4. Well defined service areas for given transmitter power.

Disadvantages of FM:

1. Much more Bandwidth (as much as 20 times as much).
2. More complicated receiver and transmitter.

Applications:

Some of the applications of the FM modulation are listed below:

I. FM Radio,(88-108 MHz band, 75 kHz,)

II. TV sound broadcast, 25 kHz,

III. 2-way mobile radio, 5 kHz / 2.5 kHz.

Additional Examples:

Example 5.13: An FM wave is defined below.

$$S(t) = 12 \sin(6\text{x}10^8 \pi t + 5 \sin 1250 \pi t)$$

Find the carrier and modulating frequencies, the modulating index, and the maximum deviation of the FM wave. Also find the bandwidth of the FM wave. What power will the FM wave dissipate in a 10 ohm resistor?

Solution: From equation 5.12, we have

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]$$

Comparing with the given FM wave,

Carrier frequency = $3\text{x}10^8$ Hz = 300 MHz

Modulating signal frequency, f_m = 625 Hz

Modulation Index, β = 5 ;

Maximum frequency deviation, Δf = β f_m = 3125 Hz.

Using Carson's rule, Bandwidth = 2(3125 + 625) = 7500 Hz

Power dissipated across resistor = P,

$$P = \frac{(A_c)^2}{2R} = \frac{144}{20} = 7.2W$$

Example 5.14: Consider an FM signal with :

Δf = 10 kHz, f_m = 10 kHz, A_c = 10 V, f_c = 500 kHz

Compute and draw the spectrum for FM signal.

Solution:

Modulation index**,** β = 10 k / 10 k = 1;

From Bessel function Table- 5.1;

for β = 1; the coefficients are $J_0 = 0.77$, $J_1 = 0.44$, $J_2 = 0.11$, $J_3 = 0.02$.

The spectrum is defined as

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n \left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m) \right]$$

The single sided spectrum is shown in fig:Ex-5.14.

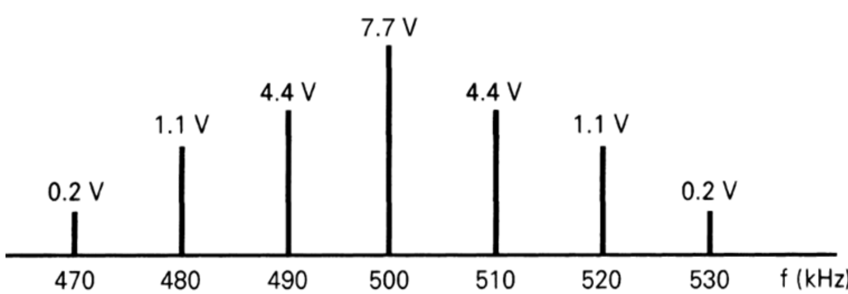


Fig: Ex-5.14 – Frequency Spectrum (for example 5.14)