UNIT 4

Vestigial side band Modulation

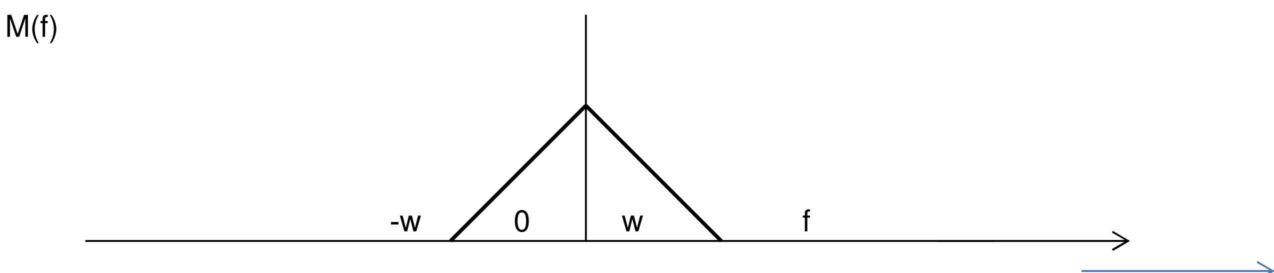
Vestigial sideband is a type of Amplitude modulation in which one side band is completely passed along with trace or tail or vestige of the other side band. VSBis a compromise between SSB and DSBSC modulation. In SSB, we send only one side band, the Bandwidth required to send SSB wave is w.

- SSB is not appropriate way of modulation when the message signal contains significant components at extremely low frequencies.

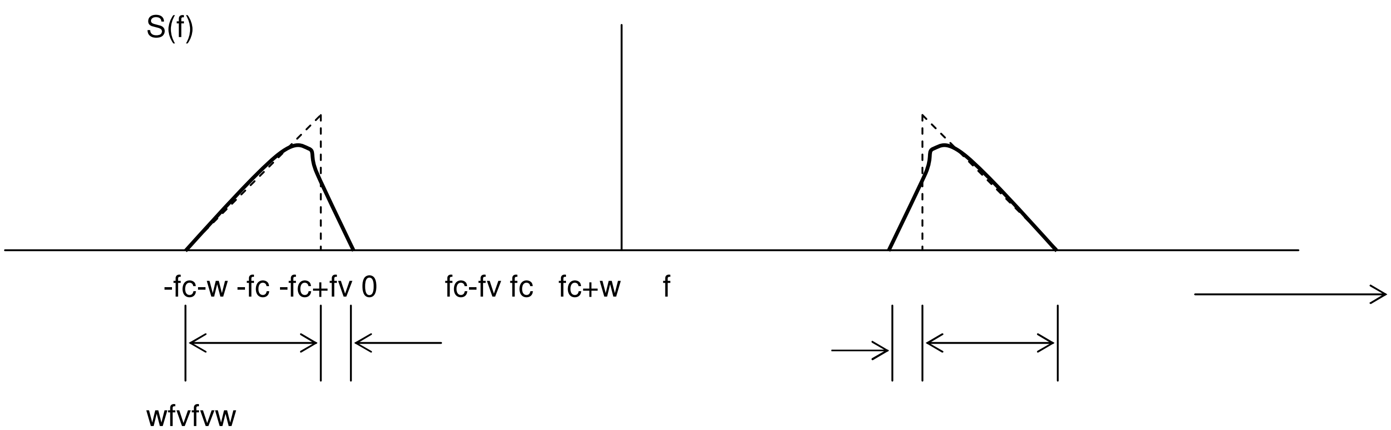
  To overcome this VSB is used

Frequency domain Description:

Fig illustrates the spectrum of VSB modulated wave s(t) with respect to the message m(t)(bandlimited)



Fig(a) Spectrum of message signal



Fig(b) Spectrum of VSB wave containing vestige of the Lower side band

Assume that the Lower side band is modified into the vestigial side band. The vestige of the lower sideband compensates for the amount removed from the upper sideband. The bandwidth required to send VSB wave is

$B = w+f_v$

Where f_v is the width of the vestigial side band.

Similarly, if Upper side band is modified into the vestigial side band then,

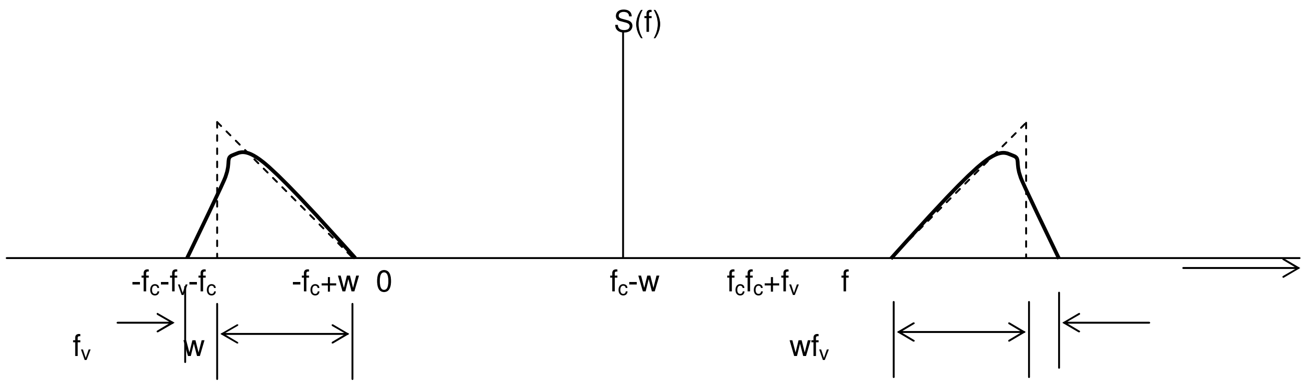


Fig (d) Spectrum of VSB wave containing vestige of the Upper side band

The vestige of the Upper sideband compensates for the amount removed from the Lower sideband. The bandwidth required to send VSB wave is

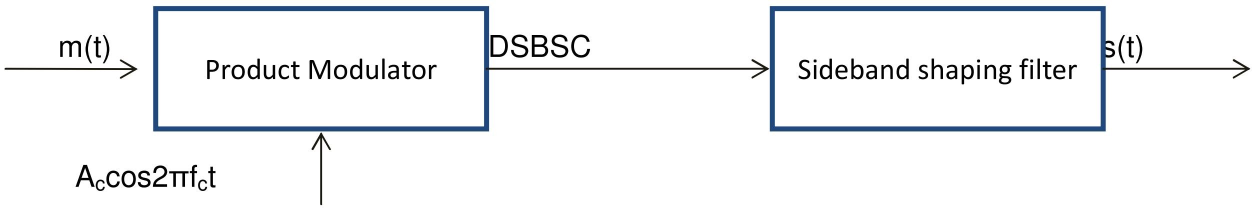
$B = w+f_v$

Where f_v is the width of the vestigial side band.

Therefore, VSB has the virtue of conserving bandwidth almost as efficiently as SSB modulation, while retaining the excellent low-frequency base band characteristics of DSBSC and it is standard for the transmission of TV signals.

Generation of VSB modulated wave:

VSB modulated wave is obtained by passing DSBSC through a sideband shaping filter as shown in fig below.



Fig(a) Block diagram of VSB Modulator

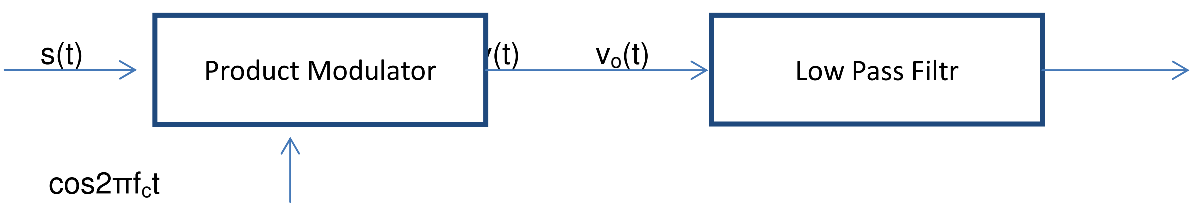
The exact design of this filter depends on the spectrum of the VSB waves. The relation b/n filter transfer function H(f) and the spectrum of VSB waves is given by

$$S(f) = A_c/2\,[M(f - f_c) + M(f + f_c)]H(f) \text{ -------------------------(1)}$$

Where M(f) is the spectrum of Message Signal.

Now, we have to determine the Specification for the Filter transfer function H(f)

It can be obtained by passing s(t) to a coherent detector and determining the necessary condition for Undistorted version of the message signal m(t). Thus,s(t) is multiplied by a Locally generated sinusoidal wave $\cos 2\pi f_c t$, which is synchronous with the carrier wave $A_c\cos 2\pi f_c t$ in both frequency and phase, as in fig below,



Fig(b). Block diagram of VSB Demodulator

Then, $v(t) = s(t).\cos 2\pi f_c t$-------------------------------------(2)

In frequency domain Eqn (2) becomes,

$$V(f) = ½\,[S(f - f_c) + S(f + f_c)] \text{ --------------------------------------(3)}$$

Substitution of Eqn (1) in Eqn (3) gives

$$V(f) = ½[A_c/2\,[M(f - f_c - f_c) + M(f - f_c + f_c)]H(f - f_c)$$

$$+ ½[A_c/2\,[M(f + f_c - f_c) + M(f + f_c + f_c)]H(f + f_c)$$

$$V(f) = ½[A_c/2\,[M(f - 2f_c) + M(f)]H(f - f_c)$$

$$+ ½[A_c/2\,[M(f) + M(f + 2f_c)]H(f + f_c)$$

$V(f) = A_c/4\ M(f)[H(f - f_c) + H(f + f_c)]$

$+ A_c/4\ [M(f - 2f_c)\ H(f - f_c) + M(f + 2f_c)\ H(f + f_c)]$ --------------------------------(4)

The spectrum of V(f) as shown in fig below,

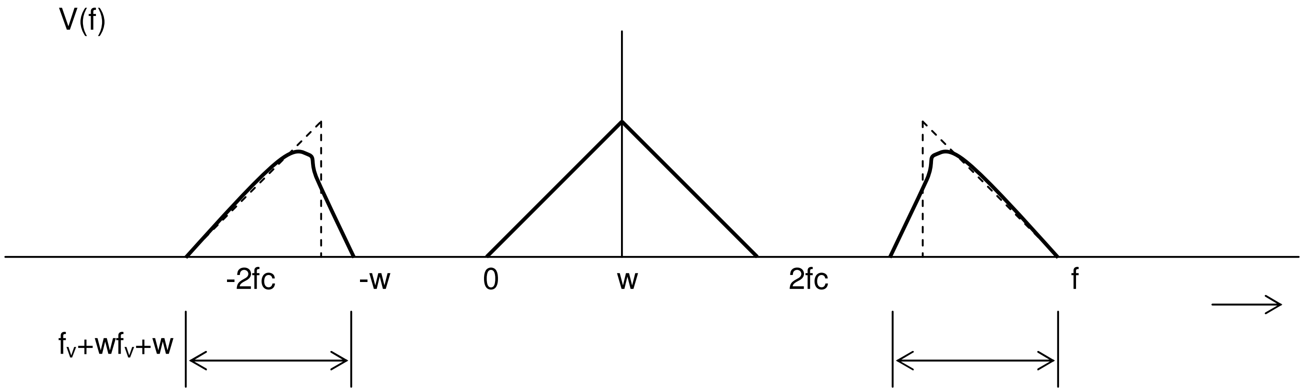


Fig ©. Spectrum of the product modulator output v(t)

Pass v(t) to a Low pass filter to eliminate VSB wave corresponding to $2f_c$.

$V_o(f) = A_c/4\ M(f)[H(f - f_c) + H(f + f_c)]$ ---(5)

The spectrum of $V_o(f)$ is in fig below,

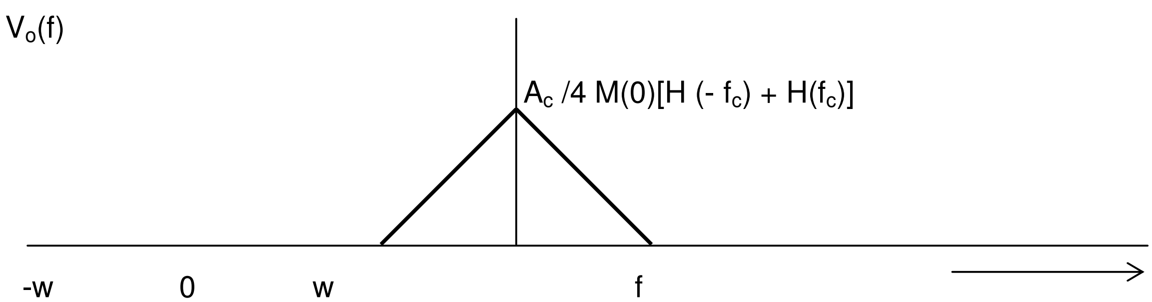


Fig (d). Spectrum of the demodulated Signal $v_o(t)$.

For a distortion less reproduction of the original signal m(t), $V_o(f)$ to be a scaled version of M(f). Therefore, the transfer function H(f) must satisfy the condition

$H(f - f_c) + H(f + f_c) = 2H(f_c)$-------------------(6)

Where $H(f_c)$ is a constant

Since m(t) is a band limited signal, we need to satisfy eqn (6) in the interval $-w \le f \le w$. The requirement of eqn (6) is satisfied by using a filter whose transfer function is shown below

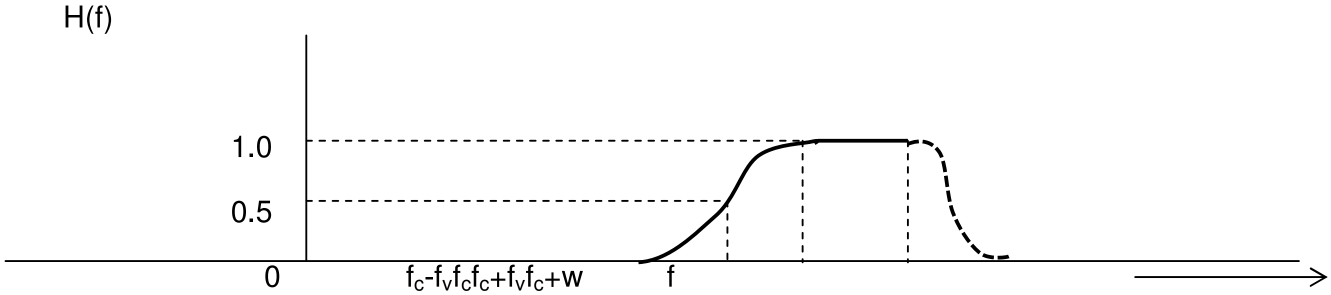


Fig (e) Frequency response of sideband shaping filter

Note: H(f) is Shown for positive frequencies only.

The Response is normalized so that H(f) at f_c is 0.5. Inside this interval $f_c-f_v \le f \le f_c+f_v$ response exhibits odd symmetry. i.e., Sum of the values of H(f) at any two frequencies equally displaced above and below is Unity.

Similarly,

The transfer function H(f) of the filter for sending Lower sideband along with the vestige of the Upper sideband is shown in fig below,

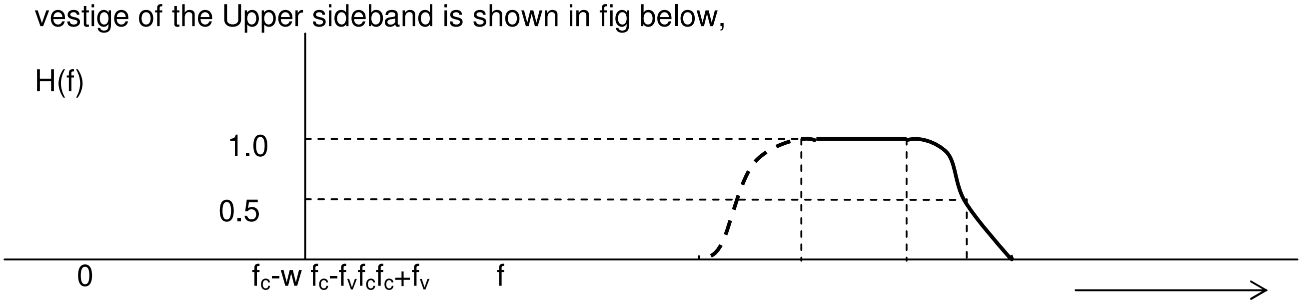


Fig (f) Frequency response of sideband shaping filter

Note: H(f) is Shown for positive frequencies only.

Time domain description:

Time domain representation of VSB modulated wave, Procedure is similar to SSB Modulated waves.

Let s(t) denote a VSB modulated wave and assuming that s(t) containing Upper sideband along with the Vestige of the Lower sideband. VSB modulated wave s(t) is the output from Sideband shaping filter, whose input is DSBSC wave. The filter transfer function H(f) is of the form as in fig below,

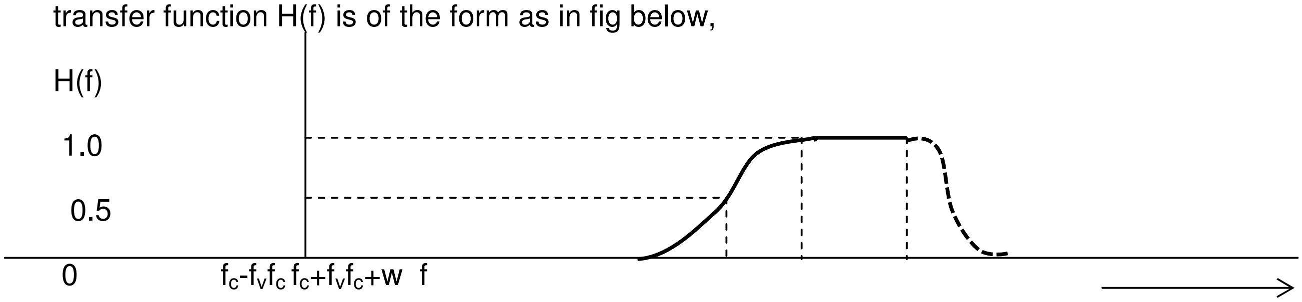


Fig (1) H(f) of sideband shaping filter

The DSBSC Modulated wave is

$S_{DSBSC}(t) = A_c\, m(t) \cos 2\pi f_c t$ ----------------------(1)

It is a band pass signal and has in-phase component only. Its low pass complex envelope is given by

$\tilde{s}_{DSBSC}(t) = A_c m(t)$ ---------------------------------------(2)

The VSB modulated wave is a band pass signal.

Let the low pass signal $\tilde{s}(t)$ denote the complex envelope of VSB wave s(t), then

$s(t) = \mathrm{Re}[\tilde{s}(t) \exp(j2\pi f_c t)]$ -------------------------(3)

To determine $\tilde{s}(t)$ we proceed as follows

1. The side band shaping filter transfer function H(f) is replaced by its equivalent complex low pass transfer function denoted by $\widetilde{H}(f)$ as shown in fig below

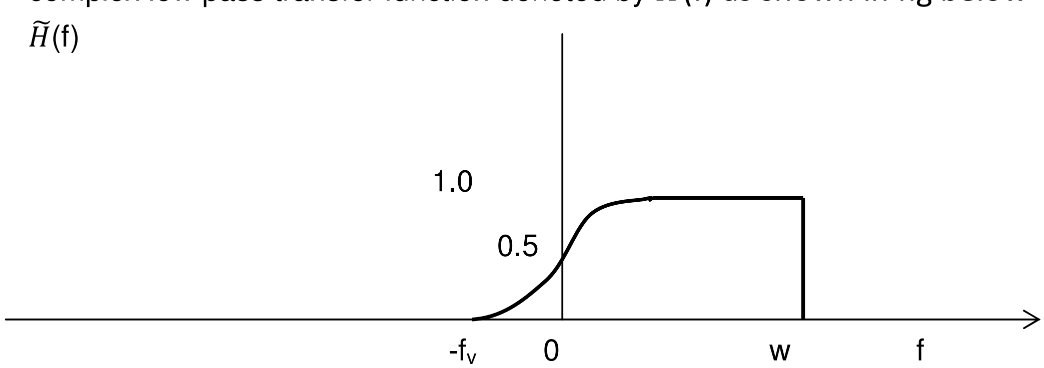


Fig (2) Low pass equivalent to H(f)

We may express $\tilde{H}$(f) as the difference between two components $\widetilde{Hu}$(f) and $\widetilde{Hv}$(f) as

$\tilde{H}$(f) = $\widetilde{Hu}$(f) - $\widetilde{Hv}$(f) ------------------------------(4)

These two components are considered individually as follows

i). The transfer function $\widetilde{Hu}$(f) pertains to a complex low pass filter equivalent to a band pass filter design to reject the lower side band completely as

$\widetilde{Hu}$(f)

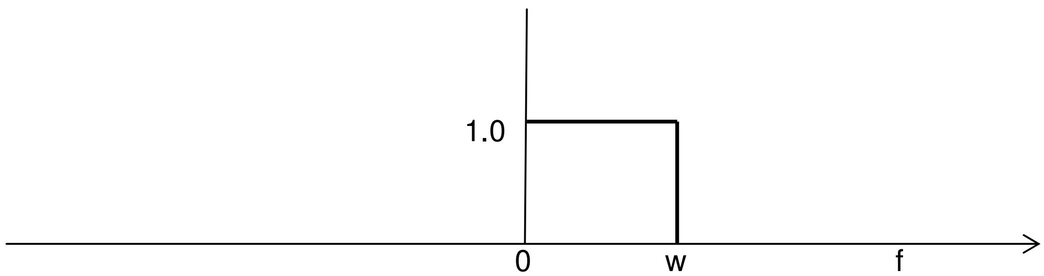


Fig (3) First component of $\tilde{H}$(f)

$$\widetilde{Hu}(f) = \begin{cases} \frac{1}{2}[1+\text{sgn}(f)], & 0<f<w \\ 0, & \text{otherwise} \end{cases} \quad \text{-----------------(5)}$$

ii). The transfer function $\widetilde{Hv}$(f) accounts for the generation of vestige and removal of a corresponding portion from the upper side band.

$\widetilde{Hv}$(f)

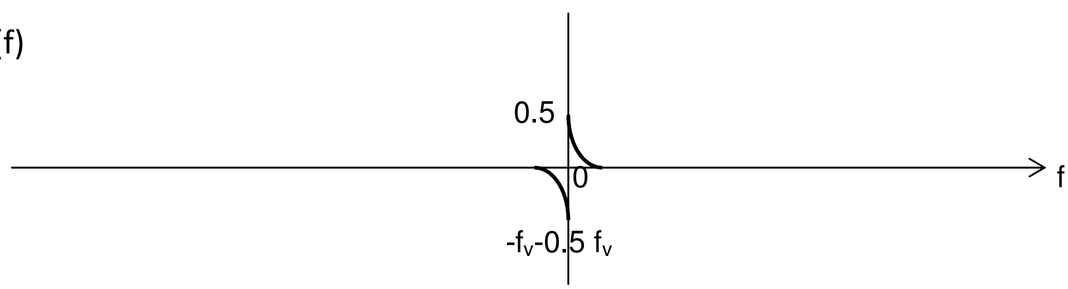


Fig (4) Second component of $\tilde{H}$(f)

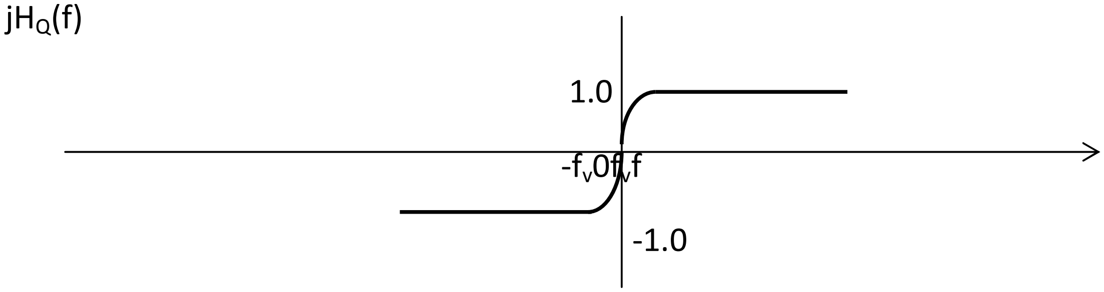
Substitute eqn(5) in eqn (4) we get,

$$\tilde{H}(f) = \begin{cases} \frac{1}{2}[1+\text{sgn}(f) - 2\widetilde{Hv}(f)], & f_v<f<w \\ 0, & \text{otherwise} \end{cases} \quad \text{-----------------(6)}$$

The sgn(f) and $\widetilde{Hv}$(f) are both odd functions of frequency, Hence, both they have purely imaginary Inverse Fourier Transform (FT). Accordingly, we may introduce a new transfer function

$H_Q(f) = 1/j[\text{sgn}(f) - 2\widetilde{Hv}(f)]$ ------------(7)

It has purely Inverse FT and $h_Q(t)$ denote IFT of $H_Q(f)$



Fig(5) Transfer function of the filter $jH_Q(f)$

Rewrite eqn(6) interms of $H_Q(f)$ as

$$\tilde{H}(f) = \begin{cases} \tfrac{1}{2}[1+jH_Q(f)], & f_v<f<w \\ 0, & \text{otherwise} \end{cases} \quad \text{-----------------(8)}$$

2. The DSBSC modulated wave is replaced by its complex envelope as
   $\tilde{S}_{DSBSC}(f) = A_c\, M(f)$ -----------------(9)
3. The desired complex envelope $\tilde{s}(t)$ is determined by evaluating IFT of the product $\tilde{H}(f)\tilde{S}_{DSBSC}(f)$.
   i.e., $\tilde{S}(f) = \tilde{H}(f)\tilde{S}_{DSBSC}(f)$ ------------------------(10)
   $\tilde{S}(f) = A_c/2[1+jH_Q(f)]\, M[f]$ ---------------------------(11)
   Take IFT of eqn(11) we get,
   $\tilde{s}(t) = A_c/2[m(t) + jm_Q(t)]$ -----------------------(12)
   Where $m_Q(t)$ is the response produced by passing the message through a low pass filter of impulse response $h_Q(t)$.
   Substitute eqn(12) in eqn(3) and simplify, we get
   $S(t) = A_c/2\ m(t) \cos 2\pi f_c t - A_c/2\ m_Q(t) \sin 2\pi f_c t$ --------------------(13)
   Where $A_c/2\ m(t)$ ----- In-phase component
   $A_c/2\ m_Q(t)$----- Quadrature component
   Note:
   1. If vestigial side band is increased to full side band, VSB becomes DSCSB.
      i.e., $m_Q(t) = 0$
   2. If vestigial side band is reduced to Zero, VSB becomes SSB.
      i.e., $m_Q(t) = \hat{m}(t)$
      Where $\hat{m}(t)$ is the Hilbert transform of m(t).

Similarly If VSB containing a vestige of the Upper sideband, then s(t) is given by

$$S(t) = A_c/2\ m(t) \cos 2\pi f_c t \ + \ A_c/2\ m_Q(t) \sin 2\pi f_c t \quad \text{--------------------(14)}$$