Unit 3

Quadrature Carrier Multiplexing

A Quadrature Carrier Multiplexing (QCM) or Quadrature Amplitude Modulation (QAM) method enables two DSBSC modulated waves, resulting from two different message signals to occupy the same transmission band width and two message signals can be separated at the receiver. The transmitter and receiver for QCM are as shown in figure 3.1.

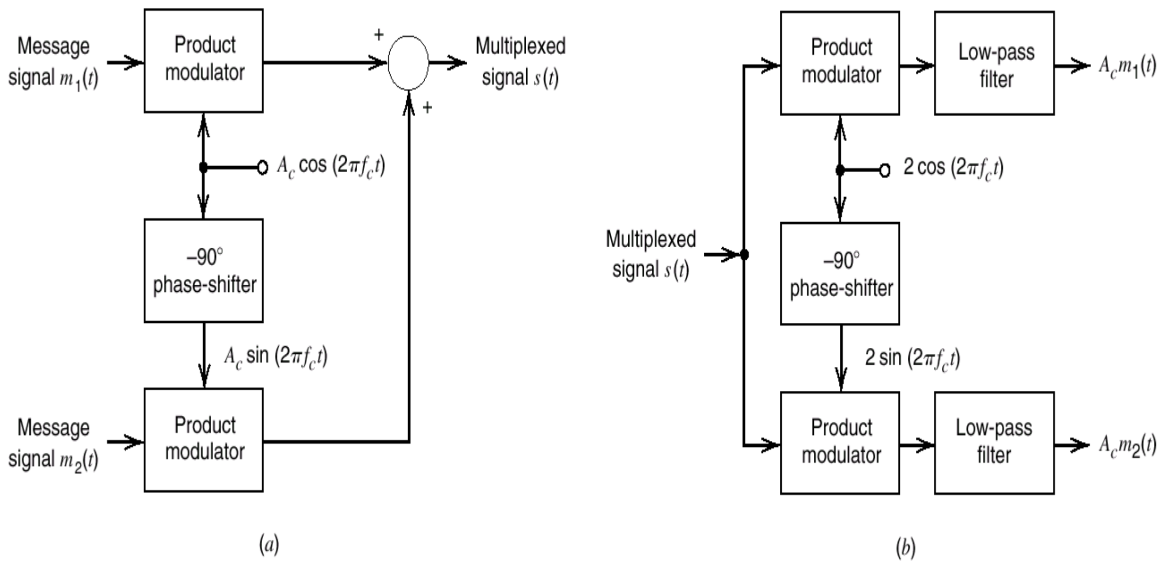


Figure 3.1: QCM transmitter and receiver

The transmitter involves the use of two separate product modulators that are supplied with two carrier waves of the same frequency but differing in phase by -90^o. The multiplexed signal *s(t)* consists of the sum of the two product modulator outputs given by the equation 3.1.

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t) \qquad \text{---------- (3.1)}$$

where $m_1(t)$ and $m_2(t)$ are two different message signals applied to the product modulators. Thus, the multiplexed signal *s(t)* occupies a transmission band width of *2W*, centered at the carrier frequency f_c where *W* is the band width of message signal $m_1(t)$ or $m_2(t)$, whichever is larger.

At the receiver, the multiplexed signal *s(t)* is applied simultaneously to two separate coherent detectors that are supplied with two local carriers of the same frequency but differing in phase by -90°. The output of the top detector is $\frac{1}{2}A_c m_1(t)$ and that of the bottom detector is $\frac{1}{2}A_c m_2(t)$.

For the QCM system to operate satisfactorily, it is important to maintain correct phase and frequency relationships between the local oscillators used in the transmitter and receiver parts of the system.

Hilbert transform

The Fourier transform is useful for evaluating the frequency content of an energy signal, or in a limiting case that of a power signal. It provides mathematical basis for analyzing and designing the frequency selective filters for the separation of signals on the basis of their frequency content. Another method of separating the signals is based on phase selectivity, which uses phase shifts between the appropriate signals (components) to achieve the desired separation.

In case of a sinusoidal signal, the simplest phase shift of 180° is obtained by "Ideal transformer" (polarity reversal). When the phase angles of all the components of a given signal are shifted by 90°, the resulting function of time is called the "Hilbert transform" of the signal.

Consider an LTI system with transfer function defined by equation 3.2.

$$H(f) = \begin{cases} -j, f > 0 \\ 0, f = 0 \\ j, f < 0 \end{cases} \qquad \text{----------------- (3.2)}$$

and the Signum function given by $$\operatorname{sgn}(f) = \begin{cases} 1, f > 0 \\ 0, f = 0 \\ -1, f < 0 \end{cases}$$

The function *H(f)* can be expressed using Signum function as given by 3.3.

$$H(f) = -j\operatorname{sgn}(f) \qquad \text{--------------------- (3.3)}$$

We know that $1e^{-j\pi/2} = -j$, $1e^{j\pi/2} = j$ and $e^{\pm j\theta} = \cos(\theta) \pm j\sin(\theta)$

Therefore,

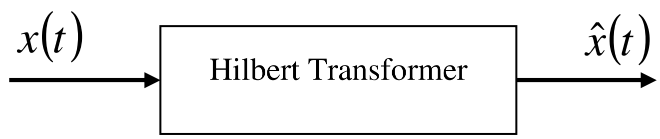
$$H(f) = \begin{cases} 1e^{-j\pi/2}, & f > 0 \\ 1e^{j\pi/2}, & f < 0 \end{cases}$$

Thus the magnitude $|H(f)| = 1$, for all f, and angle

$$\angle H(f) = \begin{cases} -\pi/2, & f > 0 \\ +\pi/2, & f < 0 \end{cases}$$

The device which possesses such a property is called Hilbert transformer. When ever a signal is applied to the Hilbert transformer, the amplitudes of all frequency components of the input signal remain unaffected. It produces a phase shift of -90^{o} for all positive frequencies, while a phase shift of 90^{o} for all negative frequencies of the signal.

If $x(t)$ is an input signal, then its Hilbert transformer is denoted by $\hat{x}(t)$ and shown in the following diagram.



To find impulse response $h(t)$ of Hilbert transformer with transfer function $H(f)$. Consider the relation between Signum function and the unit step function.

$$\operatorname{sgn}(t) = 2u(t) - 1 = x(t),$$

Differentiating both sides with respect to t,

$$\frac{d}{dt}\{x(t)\} = 2\delta(t)$$

Apply Fourier transform on both sides,

$$\operatorname{sgn}(t) \leftrightarrow \frac{2}{j\omega} \longrightarrow \operatorname{sgn}(t) \leftrightarrow \frac{1}{j\pi f}$$

Applying duality property of Fourier transform,

$$-Sgn(f) \leftrightarrow \frac{1}{j\pi t}$$

We have $$H(f) = -j\,\text{sgn}(f)$$

$$H(f) \leftrightarrow \frac{1}{\pi t}$$

Therefore the impulse response *h(t)* of an Hilbert transformer is given by the equation 3.4,

$$h(t) = \frac{1}{\pi t} \qquad \text{-------------------- (3.4)}$$

Now consider any input *x(t)* to the Hilbert transformer, which is an LTI system. Let the impulse response of the Hilbert transformer is obtained by convolving the input *x(t)* and impulse response *h(t)* of the system.

$$\hat{x}(t) = x(t) * h(t)$$

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{(t-\tau)} d\tau \qquad \text{-------------------- (3.5)}$$

The equation 3.5 gives the Hilbert transform of *x(t)*.

The inverse Hilbert transform *x(t)* is given by

$$x(t) = \frac{-1}{\pi} \int_{-\infty}^{+\infty} \frac{\hat{x}(\tau)}{(t-\tau)} d\tau \qquad \text{-------------------- (3.6)}$$

We have $\hat{x}(t) = x(t) * h(t)$

The Fourier transform $\hat{X}(f)$ of $\hat{x}(t)$ is given by

$$\hat{X}(f) = X(f)H(f)$$

$$\hat{X}(f) = -j\,\text{sgn}(f)X(f) \qquad \text{-------------------- (3.7)}$$

Applications of Hilbert transform

1. It is used to realize phase selectivity in the generation of special kind of modulation called Single Side Band modulation.

2. It provides mathematical basis for the representation of band pass signals.

Note: Hilbert transform applies to any signal that is Fourier transformable.

Example: Find the Hilbert transform of $x(t) = \cos(2\pi f_c t)$

$$X(f) = \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

$$\hat{X}(f) = -j\,\mathrm{sgn}(f)X(f)$$

$$\hat{X}(f) = -j\,\mathrm{sgn}(f)\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

$$\hat{X}(f) = \frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$$

$$\hat{X}(f) = \frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$$

This represents Fourier transform of the sine function. Therefore the Hilbert transform of cosine function is sin function given by

$$\hat{x}(t) = \sin(2\pi f_c t)$$

Pre-envelope

Consider a real valued signal *x(t)*. The pre-envelope $x_+(t)$ for positive frequencies of the signal *x(t)* is defined as the complex valued function given by equation 3.8.

$$x_+(t) = x(t) + j\hat{x}(t) \qquad \text{------------------ (3.8)}$$

Apply Fourier transform on both the sides,

$$X_+(f) = X(f) + j[-j\,\mathrm{sgn}(f)X(f)]$$

$$X_+(f) = \begin{cases} 2X(f), f > 0 \\ X(0), f = 0 \\ 0, f < 0 \end{cases} \qquad \text{-------------------- (3.9)}$$

The pre-envelope $x_-(t)$ for negative frequencies of the signal is given by

$$x_-(t) = x(t) - j\hat{x}(t)$$

The two pre-envelopes $x_+(t)$ and $x_-(t)$ are complex conjugate of each other, that is $x_+(t) = x_-(t)^*$

The spectrum of the pre-envelope $x_+(t)$ is nonzero only for positive frequencies as emphasized in equation 3.9. Hence plus sign is used as a subscript. In contrast, the spectrum of the other pre-envelope $x_-(t)$ is nonzero only for negative frequencies. That is

$$X_-(f) = \begin{cases} 0, f > 0 \\ X(0), f = 0 \\ 2X(f), f < 0 \end{cases} \qquad \text{------------------- (3.10)}$$

Thus the pre-envelopes $x_+(t)$ and $x_-(t)$ constitute a complementary pair of complex valued signals.

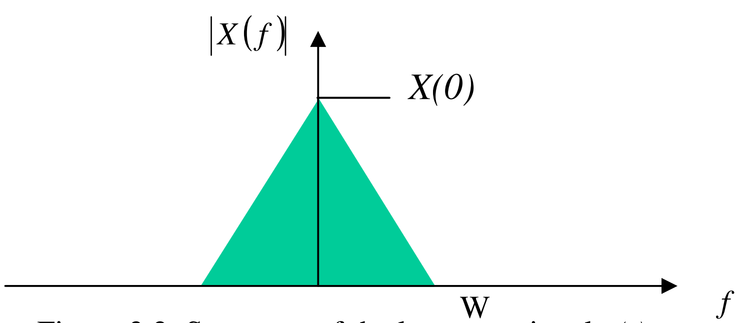


Figure 3.2: Spectrum of the low pass signal *x(t)*

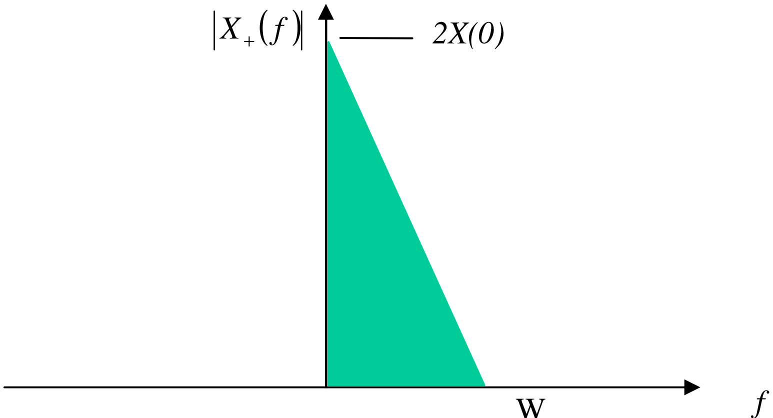


Figure 3.3: Spectrum of pre-envelope $x_+(t)$

Properties of Hilbert transform

1. "A signal *x(t)* and its Hilbert transform $\hat{x}(t)$ have the same amplitude spectrum".

The magnitude of *–jsgn(f)* is equal to 1 for all frequencies *f*. Therefore *x(t)* and $\hat{x}(t)$ have the same amplitude spectrum.

That is $$|\hat{X}(f)| = |X(f)| \qquad \text{for all } f$$

2. "If $\hat{x}(t)$ is the Hilbert transform of *x(t)*, then the Hilbert transform of $\hat{x}(t)$, is *–x(t)*".

To obtain its Hilbert transform of *x(t)*, *x(t)* is passed through a LTI system with a transfer function equal to *–jsgn(f)*. A double Hilbert transformation is equivalent to passing *x(t)* through a cascade of two such devices. The overall transfer function of such a cascade is equal to

$$[-j\,\mathrm{sgn}(f)]^2 = -1 \qquad \text{for all } f$$

The resulting output is *–x(t)*. That is the Hilbert transform of $\hat{x}(t)$ is equal to *–x(t)*.

Canonical representation for band pass signal

The Fourier transform of band-pass signal contains a band of frequencies of total extent *2W*. The pre-envelope of a narrow band signal *x(t)* is given by

$$x_+(t) = \tilde{x}(t)e^{j2\pi f_c t}, \qquad \text{---------------------- (3.11)}$$

where $\tilde{x}(t)$ is complex envelope of the signal *x(t)*.

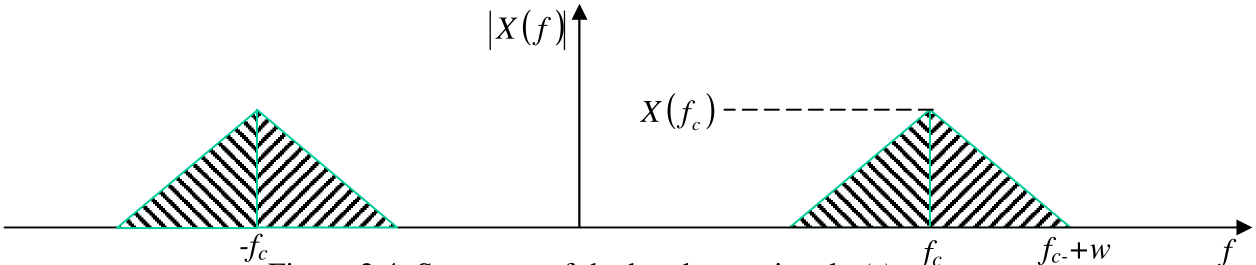


Figure 3.4: Spectrum of the band pass signal *x(t)*

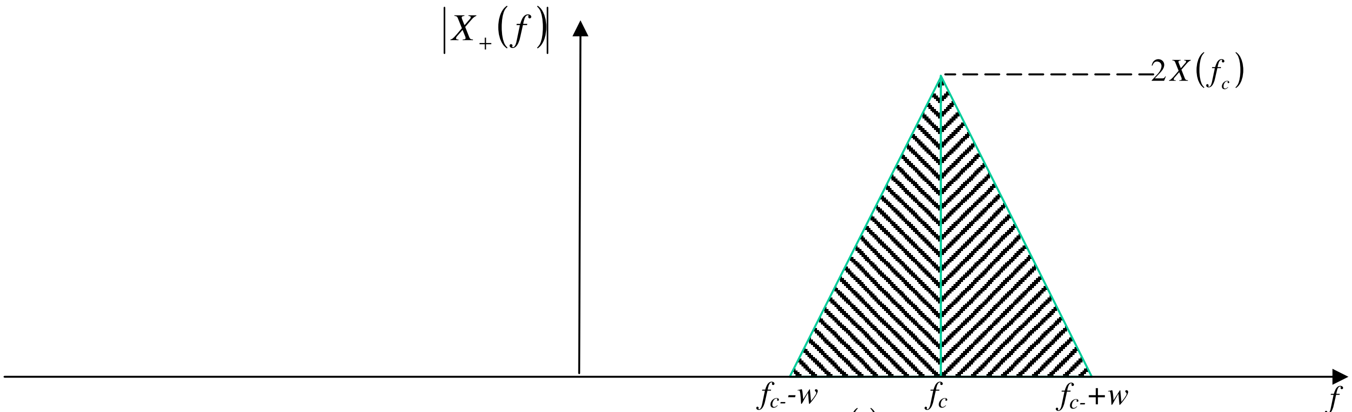


Figure 3.5: Spectrum of the pre-envelope $x_+(t)$

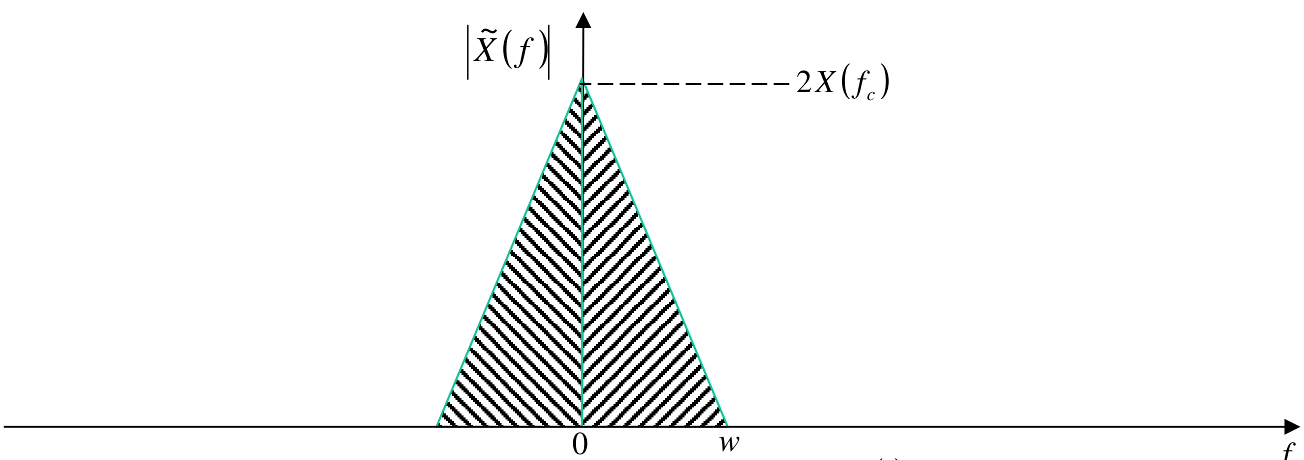


Figure 3.6: spectrum of the complex envelope $\tilde{x}(t)$

Figure 3.4 shows the amplitude spectrum of band pass signal *x(t)*. Figure 3.5 shows amplitude spectrum of pre envelope $x_+(t)$. Figure 3.6 shows amplitude spectrum of complex envelope $\tilde{x}(t)$.

Equation 3.11 is the basis of definition for complex envelope $\tilde{x}(t)$ in terms of pre-envelope $x_+(t)$. The spectrum of $x_+(t)$ is limited to the frequency band

$f_c - w < f < f_c + w$ as shown in figure 3.5. Therefore, applying the frequency-shift property of Fourier transform to equation 3.11, we find that the spectrum of the complex envelope $\tilde{x}(t)$ is limited to the band $-w < f < w$ and centered at the origin as shown in figure 3.6. That is, the complex envelope $\tilde{x}(t)$ of a band pass signal *x(t)* is a low-pass signal.

Given signal *x(t)* is the real part of the pre-envelope $x_+(t)$. So, we express the original band pass signal *x(t)* in terms of the complex envelope $\tilde{x}(t)$, as follows

$$x(t) = \text{Re}\left[\tilde{x}(t)e^{j2\pi f_c t}\right] \quad \text{------------------- (3.12)}$$

But $\tilde{x}(t)$ is a complex quantity.

$$\tilde{x}(t) = x_I(t) + jx_Q(t) \quad \text{------------------- (3.13)}$$

where $x_I(t)$ and $x_Q(t)$ are both real valued low pass functions. This low pass property is inherited from the complex envelope $\tilde{x}(t)$. Therefore original band pass signal *x(t)* is expressed in canonical form by using equations 3.12 and 3.13 as follows.

$$x(t) = \text{Re}\left[\tilde{x}(t)e^{j2\pi f_c t}\right]$$

$$x(t) = \text{Re}\left[x_I(t)e^{j2\pi f_c t} + jx_Q(t)e^{j2\pi fct}\right]$$

$$x(t) = \text{Re}\left[x_I(t)[\cos(2\pi f_c t) + j\sin(2\pi f_c t)] + jx_Q(t)[\cos(2\pi f_c t) + j\sin(2\pi f_c t)]\right]$$

$$x(t) = x_I(t)\cos(2\pi f_c t) - x_Q \sin(2\pi f_c t) \quad \text{------------- (3.14)}$$

where $x_I(t)$ is "in-phase component" and $x_Q(t)$ is "quadrature component" of the band pass signal *x(t)*. This nomenclature recognizes that $\sin(2\pi f_c t)$ is in phase-quadrature with respect to $\cos(2\pi f_c t)$.

Both $x_I(t)$ and $x_Q(t)$ are low-pass signals limited to the band $-w < f < w$. Hence, except for scaling factors, they may be derived from the band pass signal *x(t)* using the block diagram shown 3.7.

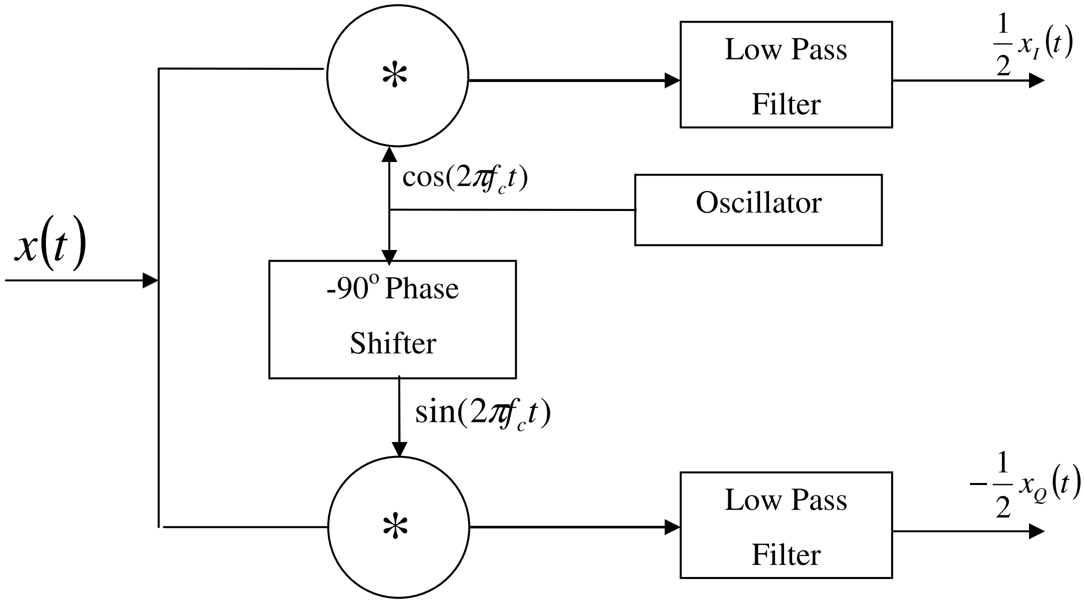


Figure 3.7: Block diagram to produce in-phase and quadrature components

Single Side Band Suppressed Carrier modulation

Standard AM and DSBSC require transmission bandwidth equal to twice the message bandwidth. In both the cases spectrum contains two side bands of width W Hz, each. But the upper and lower sides are uniquely related to each other by the virtue of their symmetry about the carrier frequency. That is, given the amplitude and phase spectra of either side band, the other can be uniquely determined. Thus if only one side band is transmitted, and if both the carrier and the other side band are suppressed at the transmitter, no information is lost. This kind of modulation is called SSBSC and spectral comparison between DSBSC and SSBSC is shown in the figures 3.8 and 3.9.

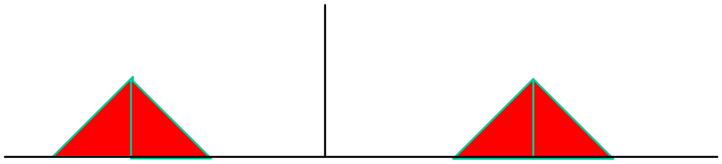


Figure 3.8: Spectrum of the DSBSC wave

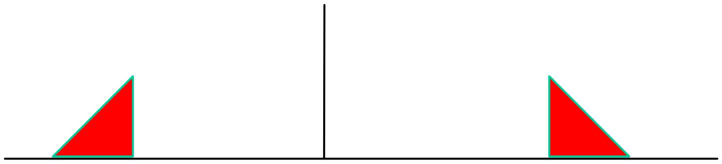


Figure 3.9: Spectrum of the SSBSC wave

Frequency-domain description: -

Consider a message signal *m(t)* with a spectrum *M(f)* band limited to the interval $-w < f < w$ as shown in figure 3.10, the DSBSC wave obtained by multiplexing *m(t)* by the carrier wave $c(t) = A_c \cos(2\pi f_c t)$ and is also shown, in figure 3.11. The upper side band is represented in duplicate by the frequencies above f_c and those below $-f_c$, and when only upper

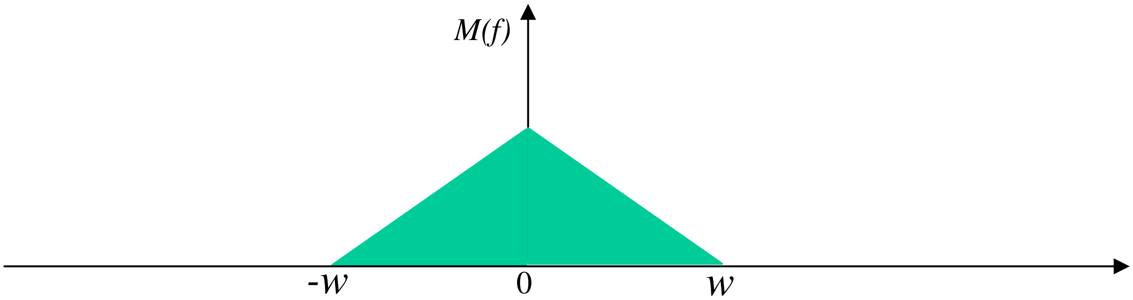


Figure 3.10: Spectrum of message wave

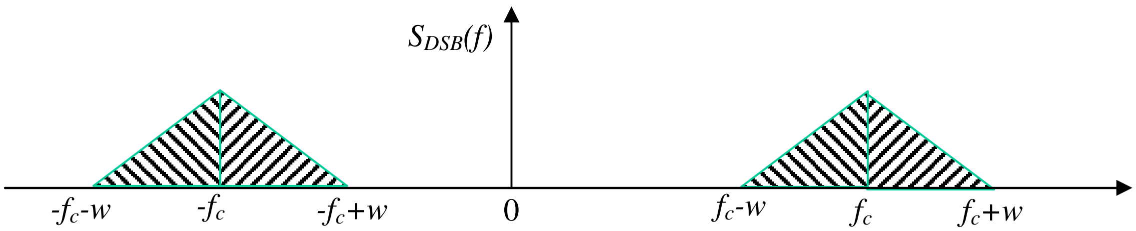


Figure 3.11: Spectrum of DSBSC wave

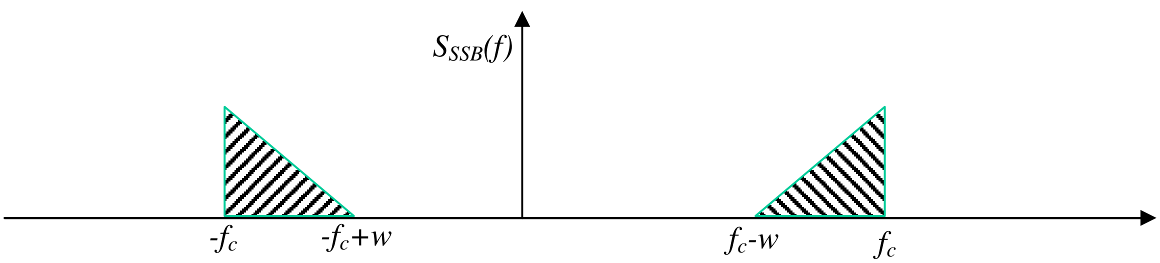


Figure 3.12: Spectrum of SSBSC-LSB wave

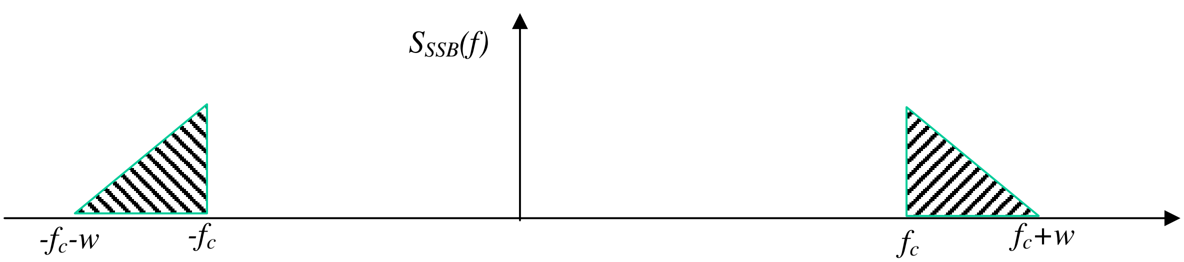


Figure 3.13: Spectrum of SSBSC-USB wave

side band is transmitted; the resulting SSB modulated wave has the spectrum shown in figure 3.13. Similarly, the lower side band is represented in duplicate by the frequencies below f_c and those above $-f_c$ and when only the lower side band is transmitted, the spectrum of the corresponding SSB modulated wave shown in figure 3.12. Thus the essential function of the SSB modulation is to translate the spectrum of the modulating wave, either with or without inversion, to a new location in the frequency domain.

The advantage of SSB modulation is reduced bandwidth and the elimination of high power carrier wave. The main disadvantage is the cost and complexity of its implementation.

Frequency Discrimination Method for generating an SSBSC modulated wave

Consider the generation of SSB modulated signal containing the upper side band only. From a practical point of view, the most severe requirement of SSB generation arises from the unwanted sideband, the nearest component of which is separated from the desired side band by twice the lowest frequency component of the message signal. It implies that, for the generation of an SSB wave to be possible, the message spectrum must have an energy gap centered at the origin as shown in figure 3.14. This requirement is naturally satisfied by voice signals, whose energy gap is about 600Hz wide.

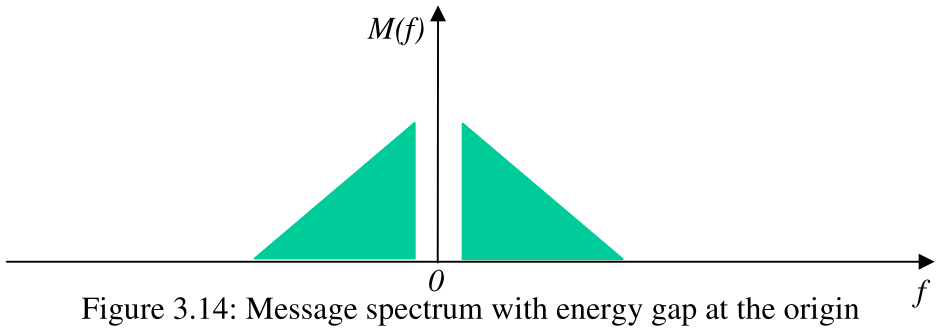
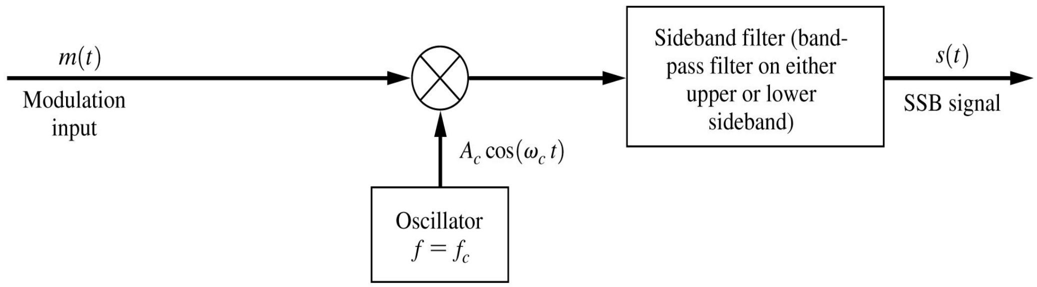


Figure 3.14: Message spectrum with energy gap at the origin

The frequency discrimination or filter method of SSB generation consists of a product modulator, which produces DSBSC signal and a band-pass filter to extract the desired side band and reject the other and is shown in the figure 3.15.



(b) Filter Method

Figure 3.15: Frequency discriminator to generate SSBSC wave

Application of this method requires that the message signal satisfies two conditions:

1. The message signal *m(t)* has no low-frequency content.

Example: - speech, audio, music.

2. The highest frequency component *W* of the message signal *m(t)* is much less than the carrier frequency f_c.

Then, under these conditions, the desired side band will appear in a non-overlapping interval in the spectrum in such a way that it may be selected by an appropriate filter. In designing the band pass filter, the following requirements should be satisfied:

1) The pass band of the filter occupies the same frequency range as the spectrum of the desired SSB modulated wave.

2. The width of the guard band of the filter, separating the pass band from the stop band, where the unwanted sideband of the filter input lies, is twice the lowest frequency component of the message signal.

When it is necessary to generate an SSB modulated wave occupying a frequency band that is much higher than that of the message signal, it becomes very difficult to design an appropriate filter that will pass the desired side band and reject the other. In such a situation it is necessary to resort to a multiple-modulation process so as to ease the filtering requirement. This approach is illustrated in the following figure 3.16 involving two stages of modulation.

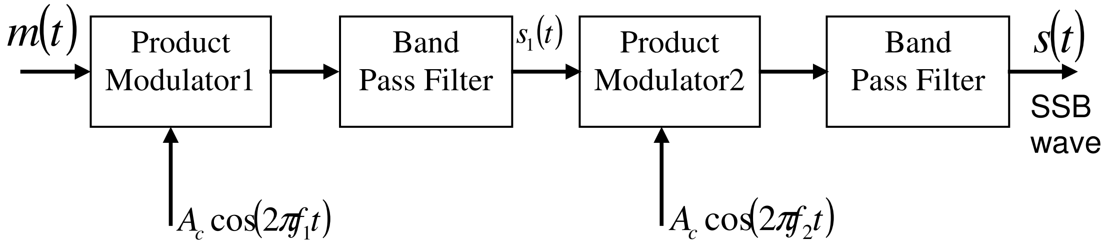


Figure 3.16: Two stage frequency discriminator

The SSB modulated wave at the first filter output is used as the modulating wave for the second product modulator, which produces a DSBSC modulated wave with a spectrum that is symmetrically spaced about the second carrier frequency f_2. The frequency separation between the side bands of this DSBSC modulated wave is effectively twice the first carrier frequency f_1, there by permitting the second filter to remove the unwanted side band.

Time-domain description

The time domain description of an SSB wave *s(t)* in the canonical form is given by the equation 3.15.

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \quad \text{-------------------- (3.15)}$$

where $S_I(t)$ is the in-phase component of the SSB wave and $S_Q(t)$ is its quadrature component. The in-phase component $S_I(t)$ except for a scaling factor, may be derived from *S(t)* by first multiplying *S(t)* by $\cos(2\pi f_c t)$ and then passing the product through a low-pass filter. Similarly, the quadrature component $S_Q(t)$, except for a scaling factor, may be derived from *s(t)* by first multiplying *s(t)* by $\sin(2\pi f_c t)$ and then passing the product through an identical filter.

The Fourier transformation of $S_I(t)$ and $S_Q(t)$ are related to that of SSB wave as follows, respectively.

$$S_I(f) = \begin{cases} S(f - f_c) + S(f + f_c), -w \le f \le w \\ 0, elsewhere \end{cases} \quad \text{------------------- (3.16)}$$

$$S_Q(f) = \begin{cases} j[S(f - f_c) - S(f + f_c)], -w \le f \le w \\ 0, elsewhere \end{cases} \quad \text{------------------- (3.17)}$$

where $-w < f < w$ defines the frequency band occupied by the message signal *m(t)*.

Consider the SSB wave that is obtained by transmitting only the upper side band, shown in figure 3.11. Two frequency shifted spectras $S(f-f_c)$ and $S(f+f_c)$ are shown in figure 3.12 and figure 3.13 respectively. Therefore, from equations 3.16 and 3.17, it follows that the corresponding spectra of the in- phase component $S_I(t)$ and the quadrature component $S_Q(t)$ are as shown in figure 3.14 and 3.15 respectively.

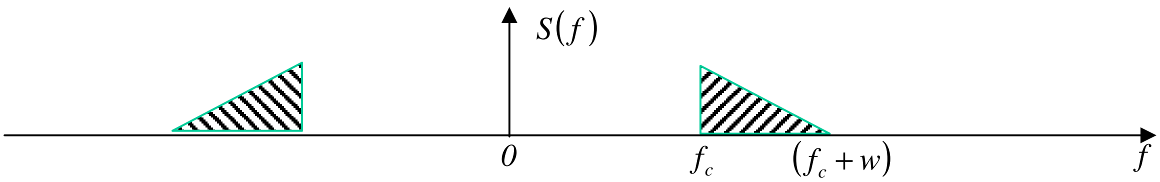


Figure 3.11: Spectrum of SSBSC-USB

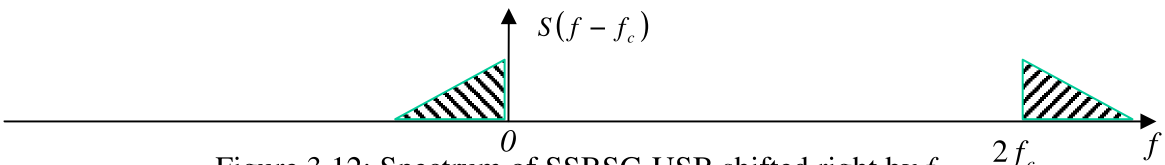


Figure 3.12: Spectrum of SSBSC-USB shifted right by f_c

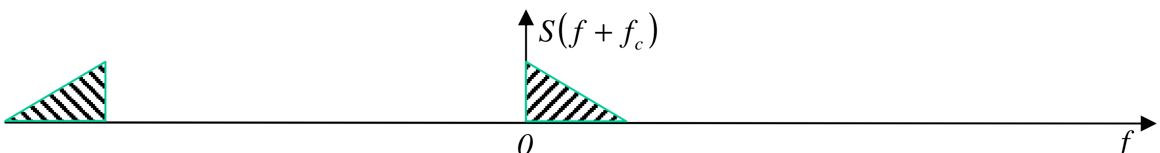


Figure 3.13: Spectrum of SSBSC-USB shifted left by f_c

Figure 3.14: Spectrum of in-phase component of SSBSC-USB

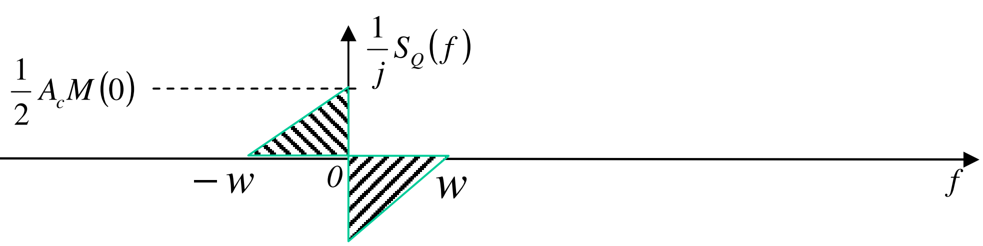


Figure 3.15: Spectrum of quadrature component of SSBSC-USB

From the figure 3.14, it is found that

$$S_I(f) = \frac{1}{2} A_c M(f)$$

where *M(f)* is the Fourier transform of the message signal *m(t)*. Accordingly in-phase component $S_I(t)$ is defined by equation 3.18.

$$s_I(t) = \frac{1}{2} A_c m(t) \qquad \text{------------------- (3.18)}$$

Now on the basis of figure 3.15, it is found that

$$S_Q(f) = \begin{cases} \frac{-j}{2} A_c M(f), & f > 0 \\ 0, & f = 0 \\ \frac{j}{2} A_c M(f), & f < 0 \end{cases}$$

$$S_Q(f) = \frac{-j}{2} A_c \operatorname{sgn}(f) M(f) \qquad \text{------------------- (3.19)}$$

where $\operatorname{sgn}(f)$ is the Signum function.

But from the discussions on Hilbert transforms, it is shown that

$$-j \operatorname{sgn}(f) M(f) = \hat{M}(f) \qquad \text{------------------- (3.20)}$$

where $\hat{M}(f)$ is the Fourier transform of the Hilbert transform of *m(t)*. Hence the substituting equation (3.20) in (3.19), we get

$$S_Q(f) = \frac{1}{2} A_c \hat{M}(f) \qquad \text{------------------------- (3.21)}$$

Therefore quadrature component $s_Q(t)$ is defined by equation 3.22.

$$s_Q(t) = \frac{1}{2} A_c \hat{m}(t) \qquad \text{-------------------------- (3.22)}$$

Therefore substituting equations (3.18) and (3.22) in equation in (3.15), we find that canonical representation of an SSB wave *s(t)* obtained by transmitting only the upper side band is given by the equation 3.23.

$$s_U(t) = \frac{1}{2} A_c m(t) \cos(2\pi f_c t) - \frac{1}{2} A_c \hat{m}(t) \sin(2\pi f_c t) \qquad \text{-------------- (3.23)}$$

Following the same procedure, we can find the canonical representation for an SSB wave *s(t)* obtained by transmitting only the lower side band is given by

$$s_L(t) = \frac{1}{2} A_c m(t)\cos(2\pi f_c t) + \frac{1}{2} A_c \hat{m}(t)\sin(2\pi f_c t) \qquad \text{-------------- (3.24)}$$

Phase discrimination method of SSB generation

Time domain description of SSB modulation leads to another method of SSB generation using the equations (3.23) or (3.24). The block diagram of phase discriminator is as shown in figure 3.16.

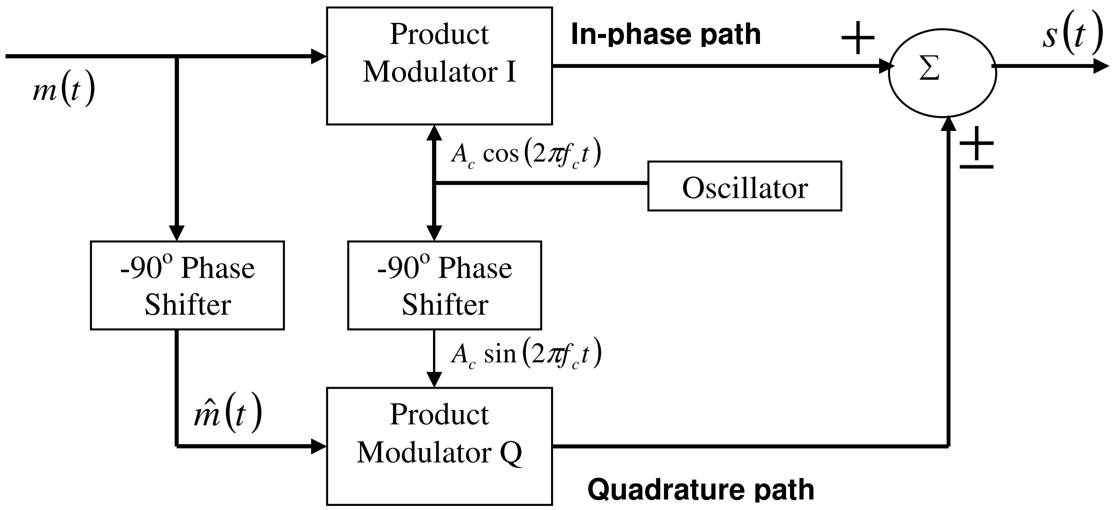


Figure 3.16: Block diagram of phase discriminator

The phase discriminator consists of two product modulators I and Q, supplied with carrier waves in-phase quadrature to each other. The incoming base band signal *m(t)* is applied to product modulator I, producing a DSBSC modulated wave that contains reference phase sidebands symmetrically spaced about carrier frequency f_c. The Hilbert transform $\hat{m}(t)$ of *m(t)* is applied to product modulator Q, producing a DSBSC modulated that contains side bands having identical amplitude spectra to those of modulator I, but with phase spectra such that vector addition or subtraction of the two modulator outputs results in cancellation of one set of side bands and reinforcement of the other set. The use of a plus sign at the summing junction yields an SSB wave with only the lower side band, whereas the use of a minus sign yields an SSB wave with only the upper side band. This modulator circuit is called Hartley modulator.

Single –tone SSB-LSB modulation

Consider a single-tone message signal $m(t) = A_m \cos(2\pi f_m t)$ and its Hilbert transform $\hat{m}(t) = A_m \sin(2\pi f_m t)$. Substituting these in equation (3.24), we get

$$s_L(t)=\frac{1}{2}A_cA_m\cos(2\pi f_m t)\cos(2\pi f_c t)+\frac{1}{2}A_cA_m\sin(2\pi f_m t)\sin(2\pi f_c t)$$

$$s_L(t)=\frac{1}{2}A_cA_m\cos(2\pi(f_c-f_m)t) \qquad \text{------------------ (3.25)}$$

Therefore the single tone SSBSC wave is a sinusoidal wave of frequency equal to sum/ difference of carrier and message frequencies for USB/LSB.

Demodulation (coherent detection) of SSBSC wave

Demodulation of SSBSC wave using coherent detection is as shown in 3.17. The SSB wave *s(t)* together with a locally generated carrier $c(t)=A_c^{1}\cos(2\pi f_c t+\phi)$ is applied to a product modulator and then low-pass filtering of the modulator output yields the message signal.

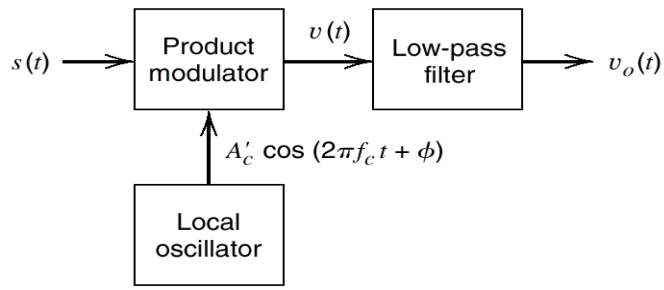


Figure 3.17: Block diagram of coherent detector for SSBSC

The product modulator output *v(t)* is given by

$$v(t)=A_c^{1}\cos(2\pi f_c t+\phi)s(t)\ , \quad \text{Put } \phi=0$$

$$v(t)=\frac{1}{2}A_c\cos(2\pi f_c t)[m(t)\cos(2\pi f_c t)\pm\hat{m}(t)\sin(2\pi f_c t)]$$

$$v(t)=\frac{1}{4}A_c m(t)+\frac{1}{4}A_c[m(t)\cos(4\pi f_c t)\pm\hat{m}(t)\sin(4\pi f_c t)] \quad \text{--- (3.26)}$$

The first term in the above equation 3.26 is desired message signal. The other term represents an SSB wave with a carrier frequency of $2f_c$ as such; it is an unwanted component, which is removed by low-pass filter.